doi：10.3969/j.issn.0490-6756.2018.02.004

单位球上 Bloch-Orlicz 空间上的复合算子

何忠华，邓 懿
(广东金融学院金融数学与统计学院，广州 510521)

摘 要：本文通过 Young 函数定义了 Bloch-Orlicz 空间，得出该空间等距同构于一类特殊的 μ-Bloch 空间. 利用复分析和构造检验函数的方法，本文研究了单位球上 Bloch-Orlicz 空间上复合算子 C_φ 的有界性、紧性是和下有界性，得到了复合算子 C_φ 是 Bloch-Orlicz 空间上的有界算子、紧性算子和下有界算子的充要条件.

关键词：复合算子；单位球；Bloch-Orlicz 空间
中图分类号：O174.5 **文献标识码**：A **文章编号**：0490-6756(2018)02-0237-06

Composition operators on Bloch-Orlicz type spaces of the unit ball

HE Zhong-Hua, *DENG Yi*
(School of Financial Mathematics and Statistics, Guangdong University of Finance, Guangzhou 510521, China)

Abstract: Using Young's functions, we define the Bloch-Orlicz space and show that the Bloch-Orlicz space is isometrically equal to a special certain μ-Bloch space. By the analysis methods and constructing test functions, we investigate the boundedness, compactness and boundedness from below of the composition operator C_φ on Bloch-Orlicz type spaces of the unit ball. And we also obtain the sufficient and necessary conditions of boundedness、compactness and boundedness from below of the composition operator C_φ.

Keywords: Composition operator; Unit ball; Bloch-Orlicz space
(2010 MSC 30H30, 46E30, 47B33)

1 Introduction

Let B^n be the unit ball in the complex vector space C^n and $H(B^n)$ the space of all analytic functions on B^n. The Bloch space[1-20] B consists of all functions $f \in H(B^n)$ for which

$$\| f \|_B := \sup_{z\in B^n}(1-|z|^2)|Rf(z)| < \infty,$$

where Rf is the radial derivative of f given by

$$Rf(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z).$$

B becomes a Banach space when it is equipp-ed with the norm[20]

$$\| f \| := |f(0)| + \| f \|_B.$$

For $\alpha > 0$, the α-Bloch space, denoted as B^α, consists of all analytic functions f on B^n such that

$$\| f \|_\alpha := \sup_{z\in B^n}(1-|z|^2)^\alpha |Rf(z)| < \infty.$$

α-Bloch spaces have been introduced and studied by numerous authors. For general theory of α-Bloch functions[19,20], many authors have studied different classes of Bloch-type spaces, where the typical weight function $w(z) = 1-|z|$ is replaced by a bounded continuous positive function μ defined on B^n. More

收稿日期：2017-09-17
基金项目：国家自然科学基金(11501130)；广东高校省级重点平台和重大科研项目基金(2015GXJK102)；广东金融学院“创新强校工程”项目基金(20170504164)
作者简介：何忠华(1984－)，男，广东南雄人，副教授，博士，主要研究泛函分析、算子理论及其代数. E-mail：zhonghuahe2010@163.com

precisely, a function $f\in H(B^n)$ is called a μ-Bloch function, denoted as $f\in B^\mu$, if

$$\|f\|_\mu:=\sup_{z\in B^n}\mu(z)|Rf(z)|<\infty.$$

Clearly, if $\mu(z)=w(z)^\alpha$ with $\alpha>0$, B^μ is just the α-Bloch space. It is readily seen that B^μ is a Banach space with the norm

$$\|f\|_{B^\mu}:=|f(0)|+\|f\|_\mu.$$

Recently, the Bloch-Orlicz type space on the unit disk was introduced by Fernandez in Ref. [9] using Young's functions. Let

$$\psi:[0,+\infty)\to[0,+\infty)$$

be a strictly increasing convex function such that $\psi(0)=0$ and $\lim\limits_{t\to+\infty}\psi(t)=+\infty$. The Bloch-Orlicz space associated with the function ψ, denoted by B^ψ, is the class of all functions $f\in H(B^n)$ such that

$$\sup_{z\in B^n}(1-|z|^2)\psi(\lambda|Rf(z)|)<\infty$$

for some $\lambda>0$ depending on f. The Minkowski's functional

$$\|f\|_\psi=\inf\{k>0:S_\psi(\frac{Rf}{k})\leq 1\}$$

defines aseminorm for B^ψ, which, in this case, is known as Luxemburg's seminorm, where

$$S_\psi(f):=\sup_{z\in B^n}(1-|z|^2)\psi(|f(z)|).$$

Moreover, B^ψ is a Banach space with the norm

$$\|f\|_{B^\psi}:=|f(0)|+\|f\|_\psi.$$

Let φ be an analytic self-map of B^n, the composition operator C_φ is the operator defined on some subspaces of $H(B^n)$ by

$$C_\varphi(f)(z):=(f\circ\varphi)(z)=f(\varphi(z)).$$

The study of composition operators thanks to Littlewood's subordination principle[4], and the function φ is called the symbol of C_φ. On composition operators between various spaces of analytic functions on different domain have been studied by numerous authors[1-3,5-11,13-16] and the other references. This paper is devoted to characterizing the boundedness, compactness and boundedness from below of composition operators on Bloch-Orlicz type space of the unit ball.

2 Auxiliary results

The following Propositions 2.1 and 2.2 are similar to Ref. [9].

Proposition 2.1 For every $f\in B^\psi\setminus\{0\}$, we have

$$S_\psi(\frac{Rf}{\|f\|_\psi})\leq 1.$$

Moreover, for any $f\in B^\psi$ and $z\in B^n$, we have

$$|Rf(z)|\leq\psi^{-1}(\frac{1}{1-|z|^2})\|f\|_\psi.$$

Proof For $f\in B^\psi\setminus\{0\}$, by the definition of B^ψ, there is a decreasing sequence $\{\lambda_k\}\subset\mathbf{R}^+$ with $S_\psi(\frac{Rf}{\lambda_k})\leq 1$ for all $k\in\mathbf{N}$, such that $\lambda_k\to\|f\|_\psi$ as $k\to\infty$. Since the function ψ is increasing, thus we have

$$S_k:=S_\psi(\frac{Rf}{\lambda_k})\leq S_\psi(\frac{Rf}{\|f\|_\psi})=:S.$$

Note that the sequence $\{S_k\}$ is increasing and bounded, then there is $S'\in\mathbf{R}$ such that

$$\lim_{k\to\infty}S_k=S'.$$

In fact, we have

$$S'=\sup_{k\in\mathbf{N}}\{S_k\}\leq 1$$

and $S'\leq S$. Furthermore, for all $z\in B^n$ and $k\in\mathbf{N}$

$$(1-|z|^2)\psi(\frac{|Rf(z)|}{\lambda_k})\leq S'.$$

Taking limit as $k\to\infty$, therefore, we have

$$(1-|z|^2)\psi(\frac{|Rf(z)|}{\|f\|_\psi})\leq S'$$

for all $z\in B^n$, which means that $S=S'=\lim\limits_{k\to\infty}S_k\leq 1$.

Moreover, by the proof above, for any $f\in B^\psi$ and $z\in B^n$, we have

$$(1-|z|^2)\psi(\frac{|Rf(z)|}{\|f\|_\psi})\leq 1.$$

Then

$$\psi(\frac{|Rf(z)|}{\|f\|_\psi})\leq\frac{1}{1-|z|^2}.$$

Since the function ψ is strictly increasing convex, thus we have

$$|Rf(z)|\leq\psi^{-1}(\frac{1}{1-|z|^2})\|f\|_\psi.$$

Proposition 2.2 The Bloch-Orlicz space is isometrically equal to μ-Bloch space, where

$$\mu(z)=\frac{1}{\psi^{-1}(\frac{1}{1-|z|^2})}$$

with $z\in B^n$.

Proof By Proposition 2.1, for any $f\in B^\psi$

and $z\in B^n$ we have $\mu(z)|Rf(z)|\le \|f\|_\psi$, which implies that $B^\psi\subset B^\mu$ and

$$\|f\|_\mu\le\|f\|_\psi.$$

Conversely, if $f\in B^\mu$, then we have

$$\mu(z)|Rf(z)|\le\|f\|_\mu$$

for all $z\in B^n$ which implies that

$$S_\psi(\frac{Rf}{\|f\|_\mu})\le 1.$$

Thus, $f\in B^\psi$ and

$$\|f\|_\psi\le\|f\|_\mu.$$

The following result will be very useful in the next section.

Lemma 2.3 Let $a\in B^n$ fixed. There exists a analytic function $f_a\in H(B^n)$ such that

$$\psi(|f_a(z)|)=\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}$$

for $z\in B^n$.

Proof For $z\in B^n$, we set

$$u(z)=\psi^{-1}(\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}).$$

Then u is a real and continuously differentiable function, in the sense that its partial derivatives exit and are continuous throughout B^n. Furthermore, for all $z\in B^n$ the function u satisfies

$$u(z)\ge\psi^{-1}(\frac{1}{4}(1-|a|^2))>0.$$

Now we let $f_a(z)=u(z)e^{iv(z)}$, where v is a real function defined on B^n. Then, in order to get f_a be an analytic function on B^n, real part $U(z)=u(z)\cos(v(z))$ and imaginary part $V(z)=u(z)\sin(v(z))$ of f_a must satisfy the Cauchy-Riemann equations. We get the relations

$$uv_x=-u_y \text{ and } uv_y=u_x.$$

And we can choose a real function $v\in C^1(B^n)$ such that f_a is an analytic function on B^n satisfying

$$\psi(|f_a(z)|)=\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}.$$

Lemma 2.4 For any $a\in B^n$, the following function is in B^ψ

$$g_a(z)=\int_0^1 f_a(tz)\frac{dt}{t}$$

with $z\in B^n$ and f_a is the function in Lemma 2.3 Moreover, $\|g_a\|_\psi=1$ for all $a\in B^n$.

Proof The result is obvious since the following equality

$$S_\psi(Rg_a):=\sup_{z\in B^n}(1-|z|^2)\frac{1-|a|^2}{|1-\langle z,a\rangle|^2}=\sup_{z\in B^n}(1-|\sigma_a(z)|^2)=1,$$

where $\sigma_a(z)=\dfrac{a-P_a(z)-S_aQ_a(z)}{1-\langle z,a\rangle}$ is the automorphism of the unit ball[20].

The following Lemma characterizes the compactness in terms of sequential convergence by the standard arguments[3].

Lemma 2.5 The composition operator C_ψ is compact on B^ψ if and only if given a bounded sequence $\{f_k\}\subset B^\psi$ such that $f_k\to 0$ uniformly on any compact subset of B^n, then $\|C_\psi(f_k)\|_\psi\to 0$ as $k\to\infty$.

3 Boundedness, compactness and boundedness from below of composition operators

In this section, we study bounedness, compactness and boundedness from below of composition operators on Bloch-Orlicz spaces.

Theorem 3.1 The composition operator C_φ is bounded on B^ψ if and only if

$$\sup_{z\in B^n}\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)|<\infty.$$

Proof Suppose that

$$L=\sup_{z\in B^n}\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)| \tag{1}$$

Then for any $f\in B^\psi\backslash\{0\}$, we have the following estimate by the Propositions 2.1 and 2.2:

$$S_\psi(\frac{R(C_\varphi f)}{L\|f\|_\psi})=$$

$$\sup_{z\in B^n}(1-|z|^2)\psi(\frac{|Rf(\varphi(z))\|R\varphi(z)|}{L\|f\|_\psi})\le$$

$$\sup_{z\in B^n}(1-|z|^2)\psi(\frac{|Rf(\varphi(z))|\mu(\varphi(z))}{\mu(z)\|f\|_\psi})\le$$

$$\sup_{z\in B^n}(1-|z|^2)\psi(\frac{\|f\|_\varphi\mu(\varphi(z))}{\mu(\psi(z))\mu(z)\|f\|_\psi})\le$$

$$\sup_{z\in B^n}(1-|z|^2)\psi(\frac{1}{\mu(z)})=1.$$

From here, we can conclude that

$$\|C_\varphi f\|\le L\|f\|_\psi$$

and thus the composition operator C_φ is bounded on B^ψ.

Conversely, suppose C_φ is bounded on B^ψ, then there exists a constant L such that $\| C_\varphi f \| \le L \| f \|_\psi$. Thus, by Lemmas 2.3 and 2.4, for any $a \in B^n$, there is a function $g_a \in B^\psi$ such that $\| g_a \|_\psi = 1$ and

$$\psi(|Rg_a(z)|) = \frac{1-|a|^2}{|1-\langle z,a\rangle|^2}.$$

Hence, for any $a \in B^n$, we have

$$(1-|z|^2)\psi(\frac{|R\varphi(z)\|Rg_a(\varphi(z))|}{L}) \le 1.$$

That is

$$(1-|z|^2)\psi(\frac{|R\varphi(z)|}{L}\psi^{-1}(\frac{1-|a|^2}{|1-\langle \varphi(z),a\rangle|^2})) \le 1.$$

Particularly, for $a = \psi(z)$, we have

$$(1-|z|^2)\psi(\frac{|R\varphi(z)|}{L}\psi^{-1}(\frac{1}{1-|\varphi(z)|^2})) \le 1.$$

Therefore

$$\frac{\mu(z)}{\mu(\varphi(z))} | R\varphi(z) | \le L.$$

From which the (1) holds.

Theorem 3.2 The composition operator C_ψ is compact on B^ψ if and only if $\psi \in B^\psi$ and

$$\lim_{|\varphi(z)|\to 1^-} \frac{\mu(z)}{\mu(\varphi(z))} | R\varphi(z) | = 0 \qquad (2)$$

Proof Suppose that $\psi \in B^\psi$ and (2) holds. Let $\{f_k\} \subset B^\psi$ be a bounded sequence such that $f_k \to 0$ uniformly on any compact subset of B^n. Then, by Lemma 2.5, it is suffices to show that $\| C_\varphi(f_k) \|_\psi \to 0$ as $k \to \infty$. To this end, we set $K = \sup_k \| f_k \|_\psi$. Then, for $\varepsilon > 0$ we can find an $r \in (0,1)$ such that

$$\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)| < \frac{\varepsilon}{K}$$

for any $z \in B^n$ satisfying $r < |\varphi(z)| < 1$. Then

$$\mu(z)|R(f_k\circ\varphi)(z)| = \frac{\mu(z)}{\mu(\psi(z))}|R\varphi(z)|\mu(\varphi(z))|Rf_k(\varphi(z))| \le \frac{\varepsilon}{K}K = \varepsilon$$

whenever $r < |\varphi(z)| < 1$.

On the other hand, since $\varphi \in B^\psi$, $\{f_k\} \subset B^\psi$ converges to 0 uniformly on any compact subset of B^n, $f_k(\varphi(0)) \to 0$ and $\sup_{|w|\le r}\mu(w)|Rf_k(w)| \to 0$ as $k \to \infty$, we can find a constant $C > 0$ depending only on r and ψ, such that

$$\sup_{|\psi(z)|\le r}\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)| \le C \| \varphi \|_\psi.$$

Therefore, for the given $\varepsilon > 0$, there is an $N \in \mathbf{N}$ such that whenever $k \ge \mathbf{N}$, we have

$$\sup_{|\psi(z)|\le r}\mu(z)|R(f_k\circ\varphi)(z)| \le \sup_{|\psi(z)|\le r}\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)|\mu(\varphi(z))|Rf_k(\varphi(z))| \le C\varepsilon \| \varphi \|_\psi.$$

Thus, we conclude thatwhenever $k \ge N$

$$\| f_k(\varphi(z)) \|_\psi = |f_k(\varphi(0))| + \sup_{z\in B^n}\mu(z)|R(C_\varphi\circ f_k)(z)| < (1+C\| \varphi \|_\psi)\varepsilon,$$

which means that C_φ is a compact operator on B^ψ.

Conversely, suppose that C_ψ is a compact operator on B^ψ but (2) doesn't hold. Then there exists an $\varepsilon_0 > 0$, such that for any $r \in (0,1)$

$$\sup_{|\psi(z)|\ge r}\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)| \ge \varepsilon_0.$$

Then, given a sequence of real number $\{r_k\} \subset (0,1)$ such that $r_k \to 1$ as $k \to \infty$, we can find a sequence $\{z_k\} \subset B^n$ such that $|\varphi(z_k)| > r_k$ and

$$\frac{\mu(z_k)}{\mu(w_k)}|R\varphi(z_k)| \ge \frac{\varepsilon_0}{2},$$

where $w_k = \varphi(z_k)$. By taking a subsequence, if necessary, we may suppose that $w_k \to w_0 \in \partial B^n$.

Now, for $k \in \mathbf{N}$ and $z \in B^n$, we set

$$g_k(z) = \int_0^1 f_{w_k}(tz)\frac{dt}{t},$$

where f_{w_k} is the function found in Lemma 2.3 with $a = w_k$. We can see that $\{g_k\}$ is a bounded sequence in B^ψ. Furthermore, we can see that $\{g_k\}$ is a sequence converging to 0 uniformly on compact subsets of B^n, and satisfying

$$\| C_\psi(g_k) \|_\mu \ge \mu(z_k)|Rg_k(w_k)\|R\varphi(z_k)| = \frac{\mu(z_k)}{\mu(w_k)}|R\varphi(z_k)| \ge \frac{1}{2}\varepsilon_0 > 0,$$

where we have used the fact that $|Rg_k(w_k)| = \frac{1}{\mu(w_k)}$. Therefore, C_ψ is not a compact operator on B^ψ, which is a contradiction.

Now we present a sufficient and necessary condition for a composition operator on B^ψ to be bounded from below (and therefore with closed range). The purpose here is to generalize the results from Refs. [1, 9, 19] for the Bloch-Orlicz space. To this end, for $\varepsilon > 0$, let us denote

$$\Omega_\varepsilon=\{z\in B^n:\frac{\mu(z)}{\mu(\varphi(z))}|R\varphi(z)|\geq\varepsilon\}.$$

Definition 3.3　A subset G of the unit ball B^n is said to be a sampling set for B^ψ if there exists a positive constant $L>0$ such that

$$\sup_{z\in G}\mu(z)|Rf(z)|\geq L\|f\|_\mu$$

for any $f\in B^\psi$.

In the following theorem we characterize-boundedness from below of composition operators on B^ψ of the ball in terms of sampling sets. The proof of the theorems follows the lines of the proofs of the corresponding results in Ref. [1].

Theorem 3.4　Let C_φ be a bounded composition operator on B^ψ. C_φ is bounded from below on B^ψ if and only if there exits $\varepsilon>0$ such that $G_\varepsilon=\varphi(\Omega_\varepsilon)$ is a sampling set for B^ψ.

Proof　Suppose that there exists $\varepsilon>0$ such that $G_\varepsilon=\varphi(\Omega_\varepsilon)$ is a sampling set for B^ψ. In this case, we can find a constant $L>0$ such that

$$\|f\|_\mu\leq L\sup_{z\in\Omega_\varepsilon}\mu(\varphi(z))|Rf(\varphi(z))|$$

for all functions $f\in B^\psi$. Thus, we have that

$$\|f\|_\mu\leq L\sup_{z\in\Omega_\varepsilon}\mu(\varphi(z))|Rf(\varphi(z))|=$$

$$L\sup_{z\in\Omega_\varepsilon}\frac{\mu(\varphi(z))}{\mu(z)|R\varphi(z)|}\mu(z)|R(f\circ\varphi)(z)|\leq$$

$$\frac{L}{\varepsilon}\|f\circ\varphi\|_\mu.$$

This implies that the operator C_φ is bounded from below on B^ψ.

Conversely, suppose that C_φ is bounded from below on B^ψ. Then there exists a constant $K>0$, such that for all $f\in B^\psi$ with $\|f\|_\mu=1$,

$$\|C_\varphi(f)\|_\mu=$$

$$\sup_{z\in B^n}\mu(z)|Rf(\varphi(z))\|R\varphi(z)|\geq K.$$

Hence, we can find $z_f\in B^n$ such that

$$\mu(z_f)|Rf(\varphi(z_f))\|R\varphi(z_f)|\geq\frac{K}{2},$$

which, in turn, implies that

$$\frac{\mu(z_f)}{\mu(\varphi(z_f))}|R\varphi(z_f)\|Rf(\varphi(z_f))|\mu(\varphi(z_f))\geq\frac{K}{2}\tag{3}$$

Since $|Rf(\varphi(z_f))|\mu(\varphi(z_f))\leq1$, it must be

$$\frac{\mu(z_f)}{\mu(\varphi(z_f))}|R\varphi(z_f)|\geq\frac{K}{2}.$$

Therefore, putting $\varepsilon=\frac{K}{2}$, we have $z_f\in\Omega_\varepsilon$.

Now, since C_φ is bounded, by Theorem 3.1, there is a constant $M_\mu>0$, depending only on μ and ψ, such that

$$\frac{\mu(z_f)}{\mu(\varphi(z_f))}|R\varphi(z_f)|\leq M_\mu.$$

From (3) we conclude that

$$|Rf(\varphi(z_f))|\mu(\varphi(z_f))\geq\frac{K}{2M_\mu}.$$

Finally, since $\varphi(z_f)\in G_\varepsilon$, it must be

$$\sup_{z\in G_\varepsilon}\mu(z)|Rf(z)|\geq\frac{K}{2M_\mu}.$$

Therefore G_ε is a sampling set for B^ψ.

References:

[1] Chen H, Gauthier P. Boundedness from below of composition operators on α-Bloch spaces[J]. Can Math Bull, 2008, 51: 195.

[2] Chen S H, Wang X. On harmonic Bloch spaces in the unit ball of $\mathbf{C}^n$[J]. B Aust Math Soc, 2011,84: 67.

[3] Cowen C C, MacCluer B D. Composition operators on spaces of analytic functions [M]. New York: CRC Press, 1995.

[4] Duren P L. Theory of H^p spaces [M]. New York: Academic Press, 1970.

[5] He Z H, Cao G F, He H. Products of integral-type operator and composition operator on Bloch-Orlicz type spaces [J]. Acta Sci Nat Univ Sunyatseni, 2016, 55: 44.

[6] Hu Z, Wang S. Composition operators on Bloch-type spaces [J]. P Edinburgh Math Soc, 2005, 5: 1229.

[7] Jiang Z J. Generalized product-type operators from weighted Bergman-Orlicz spaces to Bloch-Orlicz spaces [J]. Appl Math Comput, 2015, 268: 966.

[8] Liu L F, Cao G F, Wang X F. Composition operators on Hardy-Orlicz spaces [J]. Acta Math Sci, 2005, 25: 105.

[9] Fernandez J C R. Composition operators on Bloch-Orlicz type spaces[J]. Appl Math Comput, 2010, 217: 3392.

[10] Sehba B, Stevic S. On some product-type operators from Hardy-Orlicz and Bergman-Orlicz spaces to weighted-type spaces [J]. Appl Math Comput, 2014, 233: 565.

[11] Sharma A K, Sharma S D. Compact composition

operators on Hardy-Orlicz spaces [J]. Mat Vesnik, 2008, 60: 215.

[12] Stevic S. On new Bloch-type spaces [J]. Appl Math Comput, 2009, 215: 841.

[13] Stevic S, Chen R, Zhou Z. Weighted composition operators between Bloch type spaces in the polydisc [J]. Sb Math, 2010, 201 : 289.

[14] Wang M F, Zhao S B. Weighted composition operators between Hardy-Orlicz spaces [J]. Acta Math Sci, 2005, 25: 509.

[15] Wu S H. Bounded below property of composition operator on Bloch space in $\mathbf{C}^n$ unit ball [J]. Acta Anal Funct Appl, 2006,8: 233.

[16] Wu S H. The Essential norm of a composition operator on the Bloch space in $\mathbf{C}^n$ unit ball [J]. Acta Math Sin, 2007, 50: 1151.

[17] Yang Y, Jiang Z J. Boundedness of products of differentiation and compo-sition from weighted Bergman spaces to Zygmund spaces [J]. J Sichuan Univ: Nat Sci Ed(四川大学学报:自然科学版), 2015, 52: 731(in Chinese)

[18] Zhang K. Equivalent condition for inver-tible weighted composition opera-tors in the unit ball [J]. J Sichuan Univ: Nat Sci Ed(四川大学学报:自然科学版), 2016, 53: 989(in Chinese)

[19] Zhu K. Bloch type spaces of analytic functions [J]. Rocky MT J Math, 1993, 23: 1143.

[20] Zhu K. Spaces of holomorphic functions in the unit ball [M]. New York: Springer-Verlag, 2005.