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具有双输入时滞的网络控制系统稳定性的改进结果

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摘 要: 本文研究具有双输入时滞的网络控制系统的稳定性. 本文首先把整个时滞区间划分为三个子区间并构造了一个新的 Lyapunov 泛函,该泛函可以充分利用所有时滞的信息. 然后本文运用 Wirtinger 不等式得到了网络控制系统全局渐近稳定的充分判据. 仿真算例验证了结果的有效性. 同已有结果相比,本文的结果的保守性更低.

关键词: 稳定性; 网络控制系统; Lyapunov 泛函; 双输入时滞
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Improved results on stability analysis for a networked control system with two additive input delays

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Abstract: Stability analysis for a networked control system with two additive input delays is considered. By spitting the whole delay interval into three subintervals, a new Lyapunov-Krasovskii functional is constructed, which can make full use of the information of all the delays. Then, based on the constructed Lyapunov-Krasovskii functional and using Wirtinger-based inequality, sufficient conditions are derived to ensure the global asymptotic stability of the networked control system. Finally, a numerical example is provided to show the effectiveness of the criterion. In comparison with the known results, the obtained stability criterion is less conservative.

Keywords: Stability analysis; Networked control system; Lyapunov-Krasovskii functional; Two additive input delays

1 引 言

在实际应用中,因控制器、传感器和执行器等设备无法放在同一个位置,所以信号需要从一个地方传输到另外一个地方. 这种由通信网络连接起来的控制系统被称为网络控制系统. 与传统控制系统相比,网络控制系统具有花费少、可信度高等特点,因而引起研究者们的广泛关注[1-3].

在系统运行过程中,因放大器的切换速度是有限的,时滞产生是难以避免的. 而时滞是引起系统振

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荡、发散甚至不稳定的重要因素. 因此，时滞网络控制系统的稳定性分析成为一个研究热点[4-7].

在大多数关于时滞网络控制系统稳定性结果的报告中，状态变量中的时滞都被考虑成一个时滞[8, 9]. 然而，信号从一个点传送到另外一个点可能会经历几个阶段，网络在不同阶段的传输会产生不同性质的时滞[10]. 如图 1 所示：$d_1(t)$ 是由传感器到控制器产生的时滞；$d_2(t)$ 是由控制器到执行器产生的时滞. 当 $d_1(t)+d_2(t)$ 取到最大值的时候，并不一定有 $d_1(t)$ 和 $d_2(t)$ 同时取到最大值，从而把 $d_1(t)$ 和 $d_2(t)$ 加到一起当成一个时滞的这种处理方式是不合理的.

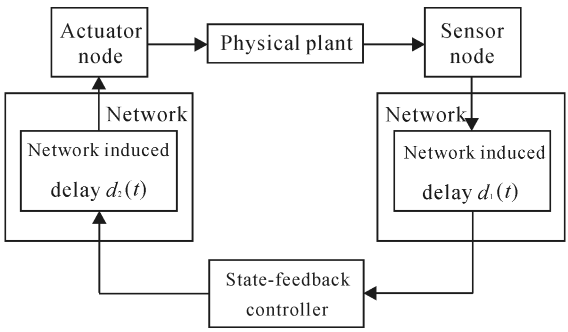

图 1　网络控制系统
Fig. 1　Networked control systems

近年来，为了能够更真实地反应动力系统的行为，一类新的、带有两个输入时滞的网络控制系统吸引了研究者的大量关注[11-15]. Lam[11] 率先研究了带有双时滞的网络系统. 紧接着，Gao[12] 对带有双时滞的网络控制系统的稳定性进行了研究. Li[14] 通过构造一个新的 Lyapunov 泛函，得到了比 Lam 和 Gao 得到的保守性更低的充分判据. Ge[15] 运用新的积分不等式，Yu[13] 则充分运用边界时滞状态得到了更进一步的改进结果.

在以上文献中，在对 Lyapunov 泛函进行估计时都是基于 Jensen 不等式放缩. 然而，众所周知，由 Wirtinger 不等式可得到比 Jensen 不等式更紧的上界，故文献[11～15]所得结果仍是保守的. 本文构造了一个新的 Lyapunov 泛函，运用 Wirtinger 不等式得到了网络控制系统全局渐近稳定的保守性更低的充分判据. 数值算例验证了结果的有效性.

2　预备知识

以上标 T 表示矩阵的转置，$\mathbf{R}^n$ 表示 n 维欧氏空间，$\mathbf{R}^{m\times n}$ 表示 $m\times n$ 的实矩阵，I 表示任意适维的单位矩阵. 对实对称矩阵 X 和 Y，$X>Y$ 表示 $X-Y$ 是正定矩阵，$0_{m,n}$ 表示 $m\times n$ 零矩阵，

$$e_i^T=(\overbrace{0,0,\cdots,}^{i-1\text{个}}I,\overbrace{0,\cdots,0}^{11-i\text{个}})(i=1,2,\ldots,11),$$

$$e_i^{*T}=(\overbrace{0,0,\cdots,}^{i-1\text{个}}I,\overbrace{0,\cdots,0}^{12-i\text{个}})(i=1,2,\ldots,12).$$

考虑下列时滞网络控制系统：

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{1}$$

其中 A 和 B 是已知常数矩阵. 时滞状态反馈控制器为 $u(t)=Kx(t-d(t))$，这里 $d(t)$ 是时变时滞，满足

$$0\leqslant d(t)\leqslant h,\dot{d}(t)\leqslant\mu \tag{2}$$

设

$$d(t)=d_1(t)+d_2(t) \tag{3}$$

其中

$$\begin{aligned}&0\leqslant d_1(t)\leqslant h_1,0\leqslant d_2(t)\leqslant h_2,\\&\quad\dot{d}_1(t)\leqslant\mu_1,\dot{d}_2(t)\leqslant\mu_2,\\&\quad h=h_1+h_2,\mu=\mu_1+\mu_2\end{aligned} \tag{4}$$

综上可得如下双时滞网络控制系统：

$$\dot{x}(t)=Ax(t)+BKx(t-d_1(t)-d_2(t)) \tag{5}$$

引理 2.1[16]　对任意的常矩阵 $W\in\mathbf{R}^{n\times n}$，$W=W^T>0$，参数 $h>0$，向量函数 $\dot{x}:[-h,0]\to\mathbf{R}^n$，有下列不等式成立：

$$-h\int_{t-h}^{t}\dot{x}^{\mathrm{T}}(s)W\dot{x}(s)\mathrm{d}s\leqslant\zeta^T(t)\begin{bmatrix}-W & W\\ * & -W\end{bmatrix}\zeta(t),$$

其中 $\zeta^T(t)=[x^T(t)\quad x^T(t-h)]$.

引理 2.2[17]　给定矩阵 $R\in\mathbf{R}^{n\times n}$，$R=R^T>0$. 对连续可微函数 $\omega:[a,b]\to\mathbf{R}^n$，有下列不等式成立：

$$\int_a^b\dot{\omega}^T(s)R\dot{\omega}(s)\mathrm{d}s\geq\frac{1}{b-a}(\Omega_1^{\mathrm{T}}R\Omega_1+3\Omega_2^{\mathrm{T}}R\Omega_2),$$

其中

$$\Omega_1=\omega(b)-\omega(a),$$

$$\Omega_2=\omega(b)+\omega(a)-\frac{2}{b-a}\int_a^b\omega(u)\mathrm{d}u.$$

引理 2.3[18]　设在 $\mathbf{R}^m$ 上的开集 D 内，f_1，$f_2,\ldots,f_N:\mathbf{R}^m\to\mathbf{R}$ 为正函数. 则在 D 内关于 f_i 的倒凸组合满足下列关系：

$$\min_{\{a_i\mid a_i>0,\sum_i a_i=1\}}\sum_i\frac{1}{a_i}f_i(t)=$$

$$\sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t),$$

且有

$$g_{i,j}: \mathbf{R}^m \to \mathbf{R}, g_{j,i}(t) \cong g_{i,j}(t),$$

$$\begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \geq 0.$$

3 主要结果

为方便表述,我们定义 $m_h = \min\{h_1, h_2\}$, $M_h = \max\{h_1, h_2\}$, $M_h - m_h = h_{12}$,其相应的时滞为 $d_m(t)$, $d_M(t)$,相应的时滞导数上界为 μ_m, μ_M,

$$v_1(t) = \int_{t-d_m(t)}^{t} \frac{x(s)}{d_m(t)} \mathrm{d}s,$$

$$v_2(t) = \int_{t-d(t)}^{t-d_m(t)} \frac{x(s)}{d_M(t)} \mathrm{d}s,$$

$$v_3(t) = \int_{t-m_h}^{t-d(t)} \frac{x(s)}{m_h - d(t)} \mathrm{d}s,$$

$$v_4(t) = \int_{t-m_h}^{t-d_m(t)} \frac{x(s)}{m_h - d_m(t)} \mathrm{d}s,$$

$$v_5(t) = \int_{t-d(t)}^{t-m_h} \frac{x(s)}{d(t) - m_h} \mathrm{d}s,$$

$$v_6(t) = \int_{t-M_h}^{t-d(t)} \frac{x(s)}{M_h - d(t)} \mathrm{d}s,$$

$$v_7(t) = \int_{t-d(t)}^{t-M_h} \frac{x(s)}{d(t) - M_h} \mathrm{d}s,$$

$$v_8(t) = \int_{t-h}^{t-d(t)} \frac{x(s)}{h - d(t)} \mathrm{d}s,$$

$$\eta(t) = [x^T(t) \;\; x^T(t-d_m(t)) \;\; x^T(t-d_M(t)) \;\; x^T(t-d(t)) \;\; x^T(t-m_h) \;\; x^T(t-M_h) \;\; x^T(t-h) \;\; \dot{x}^T(t)]^T,$$

$$\eta_1(t) = [\eta^T(t) \;\; v_1^T(t) \;\; v_2^T(t) \;\; v_3^T(t)]^T,$$

$$\eta_2(t) = [\eta^T(t) \;\; v_1^T(t) \;\; v_4^T(t) \;\; v_5^T(t) \;\; v_6^T(t)]^T,$$

$$\eta_3(t) = [\eta^T(t) \;\; v_1^T(t) \;\; v_4^T(t) \;\; v_7^T(t) \;\; v_8^T(t)]^T.$$

当 $h_1 \neq h_2$ 时,如下定理成立.

定理 3.1 在条件(2)~(4)下,系统(5)是全局渐近稳定的,如果对给定常数 h_1, h_2, μ_1 和 μ_2,存在矩阵 $P^T = P$, $Q_i^T = Q_i (i=4,5,6)$, $Q_i > 0 (i=1,2,3)$, $Z_i > 0 (i=1,2,3)$, $X_i (i=1,2,\cdots,9)$, $Y_i (i=1,2)$ 和 $\varepsilon_i (i=1,2)$ 使得下列线性矩阵不等式成立:

$$\Pi_1 = \begin{bmatrix} \frac{1}{3}P + Z_1 & -Z_1 \\ * & m_h Q_4 + Z_1 \end{bmatrix} > 0 \tag{6}$$

$$\Pi_2 = \begin{bmatrix} \frac{1}{3}P + Z_2 & -Z_2 \\ * & h_{12} Q_5 + Z_2 \end{bmatrix} > 0 \tag{7}$$

$$\Pi_3 = \begin{bmatrix} \frac{1}{3}P + Z_3 & -Z_3 \\ * & m_h Q_6 + Z_3 \end{bmatrix} > 0 \tag{8}$$

$$\Omega_i = \begin{bmatrix} Z_1 & X_i \\ X_i^T & Z_1 \end{bmatrix} \geq 0 (i=1,2,\cdots,9) \tag{9}$$

$$\Delta_i = \begin{bmatrix} Z_2 & Y_i \\ Y_i^T & Z_2 \end{bmatrix} \geq 0, (i=1,2) \tag{10}$$

$$\Upsilon_i = \begin{bmatrix} Z_3 & \varepsilon_i \\ * & Z_3 \end{bmatrix} \geq 0, (i=1,2) \tag{11}$$

$$\Phi_1 - m_h^{-1}(M_1^T \Gamma_1 M_1 + 3F^T \Gamma_2 F) - h_{12}^{-1} e_{5,6}^T Z_2 e_{5,6} - m_h^{-1} e_{6,7}^T Z_3 e_{6,7} < 0 \tag{12}$$

$$\Phi_2 - m_h^{-1}(M_2^T \Omega_7 M_2 + 3S_1^T \Omega_8 S_1) - h_{12}^{-1}(M_3^T \Delta_1 M_3 + 3S_2^T \Delta_2 S_2) - m_h^{-1} e_{6,7}^{*T} Z_3 e_{6,7}^* < 0 \tag{13}$$

$$\Phi_2 - m_h^{-1}(M_2^T \Omega_7 M_2 + 3S_1^T \Omega_8 S_1) - h_{12}^{-1} e_{5,6}^{*T} Z_2 e_{5,6}^* - m_h^{-1}(M_4^T \Upsilon_1 M_4 + 3S_3^T \Upsilon_2 S_3) < 0 \tag{14}$$

其中

$$\Phi_1 = e_1^T \alpha_{11} e_1 + e_1^T N_1 BK e_4 + e_4^T (N_1 BK)^T e_1 + e_1^T \alpha_{11} e_8 + e_8^T \alpha_{11}^T e_1 - (1-\mu_m) e_2^T Q_1 e_2 - (1-\mu_M) e_3^T Q_2 e_3 - (1-\mu) e_4^T Q_3 e_4 + e_5^T Q_{54} e_5 + e_6^T Q_{65} e_6 - e_7^T Q_6 e_7 + e_8^T \alpha_{88} e_8 + e_4^T (N_2 BK)^T e_8 + e_8^T N_1 BK e_4,$$

$$\Phi_2 = \begin{bmatrix} \Phi_1 & 0_{1,11} \\ * & 0 \end{bmatrix},$$

且

$$\alpha_{11} = N_1 A + A^T N_1^T + Q_1 + Q_2 + Q_3 + Q_4,$$

$$Q_{54} = Q_5 - Q_4, Q_{65} = Q_6 - Q_5,$$

$$\alpha_{88} = m_h Z_1 + h_{12} Z_2 + m_h Z_3 - N_2 - N_2^T,$$

$$\alpha_{18} = P - N_1 + A^T N_2^T,$$

$$e_{i,j} = e_i - e_j, e_{i,j}^* = e_i^* - e_j^*,$$

$$\Gamma_1 = \begin{bmatrix} Z_1 & X_1 & X_2 \\ * & Z_1 & X_3 \\ * & * & Z_1 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} Z_1 & X_4 & X_5 \\ * & Z_1 & X_6 \\ * & * & Z_1 \end{bmatrix},$$

$$F = \begin{bmatrix} e_1 + e_2 - 2e_9 \\ e_2 + e_4 - 2e_{10} \\ e_4 + e_5 - 2e_{11} \end{bmatrix}, M_1 = \begin{bmatrix} e_{1,2} \\ e_{2,4} \\ e_{4,5} \end{bmatrix},$$

$$M_2 = \begin{bmatrix} e_{1,2}^* \\ e_{2,5}^* \end{bmatrix}, M_3 = \begin{bmatrix} e_{5,4}^* \\ e_{4,6}^* \end{bmatrix}, M_4 = \begin{bmatrix} e_{6,4}^* \\ e_{4,7}^* \end{bmatrix},$$

$$S_1 = \begin{bmatrix} e_1^* + e_2^* - 2e_9^* \\ e_2^* + e_5^* - 2e_{10}^* \end{bmatrix},$$

$$S_2=\begin{bmatrix}e_5{}^{*}+e_4{}^{*}-2e_{11}{}^{*}\\ e_4{}^{*}+e_6{}^{*}-2e_{12}{}^{*}\end{bmatrix},$$

$$S_3=\begin{bmatrix}e_6{}^{*}+e_4{}^{*}-2e_{11}{}^{*}\\ e_4{}^{*}+e_7{}^{*}-2e_{12}{}^{*}\end{bmatrix}.$$

证明　构造下列 Lyapunov-Krasovskii 泛函：

$$V(t)=\sum_{i=1}^{4}V_i(t),$$

$$V_1(t)=x^T(t)Px(t),$$

$$V_2(t)=\int_{t-d_m(t)}^{t}x^T(s)Q_1x(s)\mathrm{d}s+\int_{t-d_M(t)}^{t}x^T(s)Q_2x(s)\mathrm{d}s+\int_{t-d(t)}^{t}x^T(s)Q_3x(s)\mathrm{d}s,$$

$$V_3(t)=\int_{t-m_h}^{t}x^T(s)Q_4x(s)\mathrm{d}s+\int_{t-M_h}^{t-m_h}x^T(s)Q_5x(s)\mathrm{d}s+\int_{t-h}^{t-M_h}x^T(s)Q_6x(s)\mathrm{d}s,$$

$$V_4(t)=\int_{-m_h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-M_h}^{-m_h}\int_{t+\theta}^{t}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta+\int_{-h}^{-M_h}\int_{t+\theta}^{t}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.$$

首先,我们验证上述泛函是正定的. 注意到 $Q_i>0$ $(i=1,2,3)$,由引理 2.1 可得

$$\begin{aligned}V(t)\geq{}&\frac{1}{3m_h}\int_{-m_h}^{0}x^T(t)Px(t)\mathrm{d}\theta+\frac{1}{3h_{12}}\int_{-M_h}^{-m_h}x^T(t)Px(t)\mathrm{d}\theta+\frac{1}{3m_h}\int_{-h}^{-M_h}x^T(t)Px(t)\mathrm{d}\theta+\\&\int_{-m_h}^{0}x^T(t+\theta)Q_4x(t+\theta)\mathrm{d}\theta+\int_{-M_h}^{-m_h}x^T(t+\theta)Q_5x(t+\theta)\mathrm{d}\theta+\int_{-h}^{-M_h}x^T(t+\theta)Q_6x(t+\theta)\mathrm{d}\theta+\\&\frac{1}{m_h}\int_{-m_h}^{0}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\begin{bmatrix}Z_1&-Z_1\\ *&Z_1\end{bmatrix}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta+\\&\frac{1}{h_{12}}\int_{-M_h}^{-m_h}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\begin{bmatrix}Z_2&-Z_2\\ *&Z_2\end{bmatrix}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta+\\&\frac{1}{m_h}\int_{-h}^{-M_h}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\begin{bmatrix}Z_3&-Z_3\\ *&Z_3\end{bmatrix}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta=\\&\frac{1}{m_h}\int_{-m_h}^{0}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\Pi_1\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta+\\&\frac{1}{h_{12}}\int_{-M_h}^{-m_h}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\Pi_2\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta+\\&\frac{1}{m_h}\int_{-h}^{-M_h}\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}^T\Pi_3\begin{bmatrix}x(t)\\x(t+\theta)\end{bmatrix}\mathrm{d}\theta.\end{aligned}$$

由式(6)～(8)可知 $V(t)$ 是正定的.

基于网络控制系统 (1),对 $V(t)$ 求导可得

$$\dot{V}_1(t)=2x^T(t)P\dot{x}(t)\tag{15}$$

$$\begin{aligned}\dot{V}_2(t)\leqslant{}&x^T(t)(Q_1+Q_2+Q_3)x(t)-(1-\mu_m)x^T(t-d_m(t))Q_1x(t-d_m(t))-\\&(1-\mu_M)x^T(t-d_M(t))Q_2x(t-d_M(t))-(1-\mu)x^T(t-d(t))Q_3x(t-d(t))\end{aligned}\tag{16}$$

$$\begin{aligned}\dot{V}_3(t)={}&x^T(t)Q_4x(t)+x^T(t-m_h)(Q_5-Q_4)x(t-m_h)+\\&x^T(t-M_h)(Q_6-Q_5)x(t-M_h)-x^T(t-h)Q_6x(t-h)\end{aligned}\tag{17}$$

$$\begin{aligned}\dot{V}_4(t)={}&\dot{x}^T(t)(m_hZ_1+h_{12}Z_2+m_hZ_3)\dot{x}(t)-\int_{t-m_h}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s-\int_{t-M_h}^{t-m_h}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s-\\&\int_{t-h}^{t-M_h}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s\end{aligned}\tag{18}$$

情形 1. 当 $d(t)\leqslant m_h$ 时,由引理 2.2 可得

$$\begin{aligned}&-m_h\int_{t-m_h}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s=\\&-m_h\int_{t-d_m(t)}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s-m_h\int_{t-d(t)}^{t-d_m(t)}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s-m_h\int_{t-m_h}^{t-d(t)}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s\leqslant\\&-\frac{m_h}{d_m(t)}\zeta_1^T(t)Z_1\zeta_1(t)-\frac{3m_h}{d_m(t)}\bar{\zeta}_1^T(t)Z_1\bar{\zeta}_1(t)-\frac{m_h}{d_M(t)}\zeta_2^T(t)Z_1\zeta_2(t)-\frac{3m_h}{d_M(t)}\bar{\zeta}_2^T(t)Z_1\bar{\zeta}_2(t)-\\&\frac{m_h}{m_h-d(t)}\zeta_3^T(t)Z_1\zeta_3(t)-\frac{3m_h}{m_h-d(t)}\bar{\zeta}_3^T(t)Z_1\bar{\zeta}_3(t),\end{aligned}$$

其中

$$\zeta_1(t)=x(t)-x(t-d_m(t)),$$

$$\bar{\zeta}_1(t)=x(t)+x(t-d_m(t))-\frac{2}{d_m(t)}\int_{t-d_m(t)}^{t}x(s)\mathrm{d}s,$$

$$\zeta_2(t)=x(t-d_m(t))-x(t-d(t)),$$

$$\bar{\zeta}_2(t)=x(t-d_m(t))+x(t-d(t))-\frac{2}{d_M(t)}\int_{t-d(t)}^{t-d_m(t)}x(s)\mathrm{d}s,$$

$$\zeta_3(t)=x(t-d(t))-x(t-h_m),$$

$$\bar{\zeta}_3(t)=x(t-d(t))+x(t-h_m)-\frac{2}{h_m-d(t)}\int_{t-h_m}^{t-d(t)}x(s)\mathrm{d}s.$$

根据引理 2.3,由(9)式可得

$$-\int_{t-m_h}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s\leqslant -m_h^{-1}\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\\ \zeta_3(t)\end{bmatrix}^T\Gamma_1\begin{bmatrix}\zeta_1(t)\\ \zeta_2(t)\\ \zeta_3(t)\end{bmatrix}- 3m_h^{-1}\begin{bmatrix}\bar{\zeta}_1(t)\\ \bar{\zeta}_2(t)\\ \bar{\zeta}_3(t)\end{bmatrix}^T\Gamma_2\begin{bmatrix}\bar{\zeta}_1(t)\\ \bar{\zeta}_2(t)\\ \bar{\zeta}_3(t)\end{bmatrix}= \eta_1^T(t)(-m_h^{-1}M_1^T\Gamma_1M_1-3m_h^{-1}F^T\Gamma_2F)\eta_1(t) \tag{19}$$

由引理 2.1 得

$$-\int_{t-M_h}^{t-m_h}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s\leqslant \eta_1{}^T(t)(-h_{12}^{-1}e_{5,6}^TZ_2e_{5,6})\eta_1(t) \tag{20}$$

$$-\int_{t-h}^{t-M_h}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s\leqslant \eta_1^T(t)(-m_h^{-1}e_{6,7}^TZ_3e_{6,7})\eta_1(t) \tag{21}$$

根据系统(5),对任意适维矩阵 N_1 和 N_2,下列等式成立:

$$2[x^T(t)N_1+\dot{x}^T(t)N_2][-\dot{x}(t)+Ax(t)+BKx(t-d(t))]=0 \tag{22}$$

由式(15)~(22)可得

$$\dot{V}(t)\leqslant\eta_1^T(t)[\Phi_1-m_h^{-1}(M_1^T\Gamma_1M_1+3F^T\Gamma_2F)-h_{12}^{-1}e_{5,6}^TZ_2e_{5,6}-m_h^{-1}e_{6,7}^TZ_3e_{6,7}]\eta_1(t).$$

由条件(12)和上式可知 $\dot{V}(t)$ 是负定的.

情形 2. 当 $m_h\leqslant d(t)\leqslant M_h$ 时,类似于情形 1 的讨论,由式(9)和(10)易知

$$-\int_{t-m_h}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s=-\int_{t-d_m(t)}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s-\int_{t-m_h}^{t-d_m(t)}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s\leqslant$$

$$\eta_2^T(t)(-\frac{1}{m_h}M_2^T\begin{bmatrix}Z_1 & X_7\\ * & Z_1\end{bmatrix}M_2-\frac{3}{m_h}S_1^T\begin{bmatrix}Z_1 & X_8\\ * & Z_1\end{bmatrix}S_1)\eta_2(t) \tag{23}$$

$$-\int_{t-M_h}^{t-m_h}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s=-\int_{t-d(t)}^{t-m_h}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s-\int_{t-M_h}^{t-d(t)}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s\leqslant$$

$$\eta_2^T(t)(-\frac{1}{h_{12}}M_3^T\begin{bmatrix}Z_2 & Y_1\\ * & Z_2\end{bmatrix}M_3-\frac{3}{h_{12}}S_2^T\begin{bmatrix}Z_2 & Y_2\\ * & Z_2\end{bmatrix}S_2)\eta_2(t) \tag{24}$$

$$-\int_{t-h}^{t-M_h}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s\leqslant \eta_2^T(t)(-m_h^{-1}e_{6,7}^{*T}Z_3e_{6,7}^*)\eta_2(t) \tag{25}$$

由式(15)~(18)及式(22)~(25)可得

$$\dot{V}(t)\leqslant\eta_2^T(t)[\Phi_2-m_h^{-1}(M_2^T\Omega_7M_2+3S_1^T\Omega_8S_1)-h_{12}^{-1}(M_3^T\Delta_1M_3+3S_2^T\Delta_2S_2)-m_h^{-1}e_{6,7}^{*T}Z_3e_{6,7}^*]\eta_2(t).$$

根据条件(13),由上式可知 $\dot{V}(t)$ 负定的.

情形 3. 当 $M_h\leqslant d(t)\leqslant h$ 时,由(9)和(11)式可得

$$-\int_{t-m_h}^{t}\dot{x}^T(s)Z_1\dot{x}(s)\mathrm{d}s\leqslant \eta_3^T(t)(-\frac{1}{m_h}M_2{}^T\begin{bmatrix}Z_1 & X_7\\ * & Z_1\end{bmatrix}M_2-\frac{3}{m_h}S_1^T\begin{bmatrix}Z_1 & X_8\\ * & Z_1\end{bmatrix}S_1)\eta_3(t),$$

$$-\int_{t-M_h}^{t-m_h}\dot{x}^T(s)Z_2\dot{x}(s)\mathrm{d}s\leqslant \eta_3^T(t)(-h_{12}^{-1}e_{5,6}^{*T}Z_2e_{5,6}^*)\eta_3(t),$$

$$-\int_{t-h}^{t-M_h}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s=-\int_{t-d(t)}^{t-M_h}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s-\int_{t-h}^{t-d(t)}\dot{x}^T(s)Z_3\dot{x}(s)\mathrm{d}s\leqslant$$

$$\eta_3^T(t)(-\frac{1}{m_h}M_4^T\begin{bmatrix}Z_3 & \varepsilon_1\\ * & Z_3\end{bmatrix}M_4-\frac{3}{m_h}S_3^T\begin{bmatrix}Z_3 & \varepsilon_2\\ * & Z_3\end{bmatrix}S_3)\eta_3(t).$$

于是有

$$\dot{V}(t) \leqslant \eta_3^T(t)[\Phi_2 - m_h^{-1}(M_2^T\Omega_7 M_2 + 3S_1^T\Omega_8 S_1) - h_{12}^{-1} e_{5,6}^{*T} Z_2 e_{5,6}^{*} - m_h^{-1}(M_4^T\Upsilon_1 M_4 + 3S_3^T\Upsilon_2 S_3)]\eta_3(t).$$

由条件(14)可知 $\dot{V}(t)$ 负定的.

综上所述，$\dot{V}(t)$ 是负定的而 $V(t)$ 是正定的. 故系统(5)是全局渐近稳定的. 证毕.

注　当 $h_1 = h_2$ 时，在定理 3.1 的证明中令 $Q_5 = Z_2 = 0$ 可得与系统(5)相应的全局渐近稳定的判据.

4　仿　真

考虑系统(1)，其参数为[11-15]：

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$K = -\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \dot{d}_1(t) \leqslant 0.1, \dot{d}_2(t) \leqslant 0.8.$$

表 1　h_2 上界最大值在不同 h_1 取值下的比较

Tab. 1　Comparison of the admissible upper bound h_2 for varied h_1

方法	h_1	1	1.2	1.5
文献[11]		0.415	0376	0248
文献[12]		0.512	0.406	0.283
文献[14]	h_2	0.583	0.519	0.421
文献[15]		0.873	0.673	0.373
文献[13]		0.928	0.789	0.459
定理 3.1		0.989	0.837	0.577

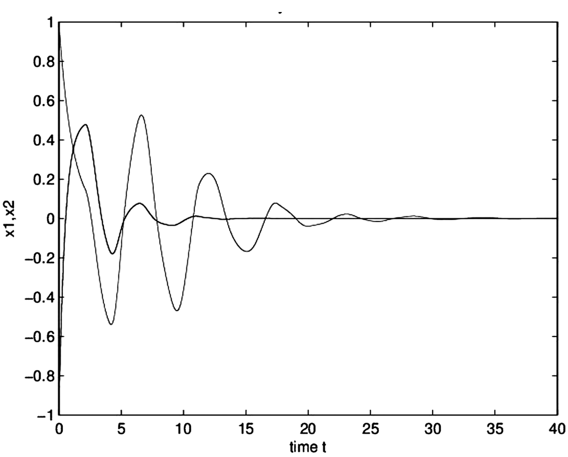

图 2　$x(t)$ 的轨迹

Fig. 2　Trajectories of $x(t)$

表 1 给出了不同文献中的方法在 h_1 不同取值情况下所对应 h_2 上界的最大值的比较. 从表 1 可以发现，本文方法所得到的结果比文献[11～15]的保守性更低. 取 $h_1 = 1.5, h_2 = 0.577$，初值 $x(0) = [-1,1]$. 图 2 演示了系统(1)在上述参数下的状态变量轨迹. 从图 2 可以看出，系统(1)在上述参数下是渐近稳定性的. 这验证了定理 3.1 的有效性.

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