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α-Bloch-Orlicz 空间复合算子与积分算子乘积的差分

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摘 要：本文研究了单位圆盘上的 α-Bloch-Orlicz 空间复合算子与积分算子的乘积 $C_\varphi I_g$ 和 $I_g C_\varphi$ 间的差分. 通过构造不同的检测函数，本文给出了判断差分的有界性和紧性的充要条件.

关键词：差分；复合算子；积分算子；α-Bloch-Orlicz 空间
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Difference of products of composition operator and integral type operator on α-Bloch-Orlicz space

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Abstract: In this paper, we study the difference of the products of composition operator and integral type operator $C_\varphi I_g$, $I_g C_\varphi$ acting on the α-Bloch-Orlicz space B_α^ϕ in the unit disk. By constructing different test functions, we present some necessary and sufficient conditions for the boundedness and compactness of the difference.

Keywords: Difference; Composition operator; Integral type operator; α-Bloch-Orlicz space
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1 Introduction

Let D be the unit disk of the complex plane **C** and $H(D)$ be the space of all analytic functions on D. Denote by $S(D)$ the collection of all holomorphic self-maps of D. Every $\varphi \in S(D)$ induces the composition operator C_φ defined as

$$C_\varphi f(z) = f \circ \varphi(z), f \in H(D), z \in D.$$

The theory of composition operator on various setting has quite a long and rich history. We can refer to Ref. [1] for various properties on the composition operators acting on different classical holomorphic function spaces.

Let μ be a weight, that is, μ is a positive continuous function on D. We recall that the μ-Bloch space B_μ consists of all $f \in H(D)$ such that

$$\|f\|_{B_\mu} = f(0) + \sup_{z \in D} \mu(z) |f'(z)| < \infty.$$

It is a well-known fact that the μ-Bloch space B_μ is a Banach space under norm $\|f\|_{B_\mu}$. In particular, when $\mu(z) = (1-|z|^2)^\alpha$ for $0<\alpha<\infty$, the induced space B_μ is α-Bloch space which can be defined as:

$$B_\alpha = \{f \in H(D): \|f\|_{B_\alpha} = \sup_{z \in D} (1-|z|^2)^\alpha |f'(z)| < \infty\}.$$

It is a Banach space endowed with the norm $\|f\|_\alpha = f(0) + \|f\|_{B_\alpha}$.

Let $g \in H(D)$. For $f \in H(D)$, the integral

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type operators I_g and J_g is defined by

$$I_g f(z)=\int_0^z f'(\xi)g(\xi)\mathrm{d}\xi$$

and

$$J_g f(z)=\int_0^z f(\xi)g'(\xi)\mathrm{d}\xi.$$

The product of composition operator and integral type operator is first introduced and discussed by Li and Stević[2,3], which can be defined by

$$(C_\varphi I_g f)(z)=\int_0^{\varphi(z)} f'(\xi)g(\xi)\mathrm{d}\xi,$$

$$(C_\varphi J_g f)(z)=\int_0^{\varphi(z)} f(\xi)g'(\xi)\mathrm{d}\xi,$$

$$(I_g C_\varphi f)(z)=\int_0^z (f\circ\varphi)'(\xi)g(\xi)\mathrm{d}\xi,$$

or

$$(J_g C_\varphi f)(z)=\int_0^z (f\circ\varphi)(\xi)g'(\xi)\mathrm{d}\xi.$$

For the investigation for difference of composition operators on μ -Bloch space, sometimes called weighted Bloch space, we can refer to Refs. [4～8]. With the distinction to the definition of $\mu(z)$, the μ -Bloch space changes into different weighted Bloch space.

In recent years, the Bloch-Orlicz space appeares in the literature motived by the study of Hardy-Orlicz space and Bergman-Orlicz space[9-12] as a further generalization of the classical Bloch space in the unit disk. Motivated by the same spirit, for $0<\alpha<\infty$, the α -Bloch-Orlicz space on the unit disk is generalized by Liang in Ref. [13] as follows:

$$B_\alpha^\phi=\{f\in H(D):\ \sup_{z\in D}(1-|z|^2)^\alpha\phi(\lambda|f'(z)|)<\infty\}$$

for some $\lambda>0$ depending on f, where ϕ is also a Young's function. We can also assume without loss of generality that ϕ^{-1} is differentiable, and the Minkowski's functional

$$\|f\|_{\phi,\alpha}=\inf\{k>0:S_{\phi,\alpha}(\frac{f'}{k})\leq 1\}$$

defines a semi-norm on B_α^ϕ, where

$$S_{\phi,\alpha}(f):=\sup_{z\in D}(1-|z|^2)^\alpha\phi(|f(z)|).$$

B_α^ϕ becomes a Banach space with the norm

$$\|f\|_{B_\alpha^\phi}=|f(0)|+\|f\|_{\phi,\alpha}.$$

In the past decades, properties including boundedness and compactness of the difference of composition operators are studied by many authors, such as Refs. [14～18]. For some good sources of information on much of the development in the theory of integral type operators, we can refer Refs. [19～21].

In this paper, we limit our study on the difference of products of integral type and composition operators $C_\varphi I_g$ and $I_g C_\varphi$ on α -Bloch-Orlicz space B_α^ϕ on the unit disk. The paper is based on my study on the difference of composition operators on B_α^ϕ in Ref. [22].

The paper is organized as follows. Some background material and lemmas follow in Section 2. Then we characterize the boundedness and compactness of $C_\varphi I_g - C_\psi I_g$ and $C_\varphi I_g - I_g C_\psi$ on the α -Bloch-Orlicz space B_α^ϕ with $0<\alpha<\infty$ in Section 3 and Section 4.

Throughout the paper, C will denote a positive constant, it's exact value may vary from one occurrence to the other.

2　Notions and lemmas

In this section we collect some lemmas and properties to be used in later sections. To begin the discussion, we introduce some nations.

The one-to-one holomorphic function which maps D to itself called the Möbius transformation, denoted by Aut(D), with the form $\lambda\varphi_a$, where $|\lambda|=1$ and φ_a is defined by

$$\varphi_a(z)=\frac{a-z}{1-\bar{a}z},z\in D$$

for $a\in D$. We have the following identities:

$$(1-|z|^2)|\varphi_a'(z)|=1-|\varphi_a(z)|^2,$$

$$1-|\varphi_a(z)|^2=\frac{(1-|a|^2)(1-|z|^2)}{|1-\bar{a}z|^2}.$$

The pseudohyperbolic distance between $a,z\in D$ is given by $\rho(a,z)=|\varphi_a(z)|$. We know that $\rho(a,z)$ is invariant under automorphisms (see Ref. [23]) and

$$\frac{1-\rho(a,z)}{1+\rho(a,z)}\leq\frac{1-|a|^2}{1-|z|^2}\leq\frac{1+\rho(a,z)}{1-\rho(a,z)}.$$

For $\varphi\in S(D)$, the Schwarz-Pick Lemma[1] shows that $\rho(\varphi(z),\varphi(w))\leq\rho(z,w)$.

Now let us state a couple of lemmas. Some of which are direct statements of Ref. [10].

Proposition 2.1[13] For $\alpha>0$,

$$S_{\phi,\alpha}\left(\frac{f'}{\|f\|_{B_\alpha^\phi}}\right)\le S_{\phi,\alpha}\left(\frac{f'}{\|f\|_{\phi,\alpha}}\right)\le 1$$

holds for each $f\in B_\alpha^\phi$.

Proof The proof is similar to Lemma 2 in Ref. [24].

***Remark* 1** For each $f\in B_\alpha^\phi$, Proposition 2.1 allows us to observe that

$$|f'(z)|\le\phi^{-1}\left(\frac{1}{(1-|z|^2)^\alpha}\right)\|f\|_{\phi,\alpha}\le \phi^{-1}\left(\frac{1}{(1-|z|^2)^\alpha}\right)\|f\|_{B_\alpha^\phi},z\in D.$$

In addition, for $z\in D$ fixed and any $f\in B_\alpha^\phi$, we have

$$|f(z)|\le|f(0)|+\int_0^z|f'(s)|\mathrm{d}s\le \left(1+|z|\phi^{-1}\left(\frac{1}{(1-|z|^2)^\alpha}\right)\right)\|f\|_{B_\alpha^\phi}$$

by integrating the aforesaid estimation.

Lemma 2.2[13] For $\alpha>0$, the α-Bloch-Orlicz space is isometrically equal to the μ_α-Bloch space, where

$$\mu_\alpha(z)=\frac{1}{\phi^{-1}\left(\frac{1}{(1-|z|^2)^\alpha}\right)}.$$

In other words,

$$\|f\|_{B_\alpha^\phi}=|f(0)|+\sup_{z\in D}\mu_\alpha(z)|f'(z)|$$

holds for each $f\in B_\alpha^\phi$.

Lemma 2.3[22] For $\alpha>0$, $f\in B_\alpha^\phi$, for any φ, $\psi\in S(D)$,

$$\left|\frac{f'(\varphi(z))}{\varphi^{-1}\left(\frac{1}{(1-|\varphi(z)|^2)^\alpha}\right)}-\frac{f'(\psi(z))}{\phi^{-1}\left(\frac{1}{(1-|\psi(z)|^2)^\alpha}\right)}\right|\le C\|f\|_{\phi,\alpha}\rho(\varphi(z),\psi(z))$$

holds for all $z\in D$.

***Remark* 2** From the proof of Lemma 2.3, it is not difficult to see that for any $\phi(z),\psi(z)\in rD=\{w\in D:|w|<r<1\}$,

$$\left|\frac{f'(\varphi(z))}{\varphi^{-1}(\frac{1}{(1-|\varphi(z)|^2)^\alpha})}-\frac{f'(\psi(z))}{\phi^{-1}(\frac{1}{(1-|\psi(z)|^2)^\alpha})}\right|\le C\|f'_r\|_{H_v^\infty}\rho(\varphi(z),\psi(z))$$

for any $f'\in H\infty_v$, where

$$\|f'_r\|_{H\infty_v}=\sup_{w\in rD}\frac{|f'(w)|}{\phi^{-1}(\frac{1}{(1-|w|^2)^\alpha})}.$$

Thus by the above argument and Remark 1, one has for any $f\in B_\alpha^\phi$

$$\left|\frac{f'(\varphi(z))}{\phi^{-1}\left(\frac{1}{(1-|\varphi(z)|^2)^\alpha}\right)}-\frac{f'(\psi(z))}{\phi^{-1}\left(\frac{1}{(1-|\psi(z)|^2)^\alpha}\right)}\right|\le C\|f'_r\|_{H_v^\infty}\rho(\varphi(z),\psi(z))= C\sup_{w\in rD}\frac{f'(w)}{\phi^{-1}\left(\frac{1}{(1-|w|^2)^\alpha}\right)}\rho(\varphi(z),\psi(z)).$$

The equivalent condition below is originally in Ref. [13].

Proposition 2.4 For $\alpha>0$, the equivalent condition

$$S_{\phi,\alpha}(f')\le 1\Leftrightarrow\|f\|_{\phi,\alpha}\le 1$$

holds for each $f\in B_\alpha^\phi$.

The construction in the following is helpful for the investigation of the boundedness of the composition operators on the α-Bloch-Orlicz space with $\alpha>0$, where the proof includes the same arguments in Ref. [24].

Lemma 2.5[22] For $\alpha>0$ and a fixed point a

$\in D$, there is a holomorphic function $f\in H(D)$ such that

$$\phi(|f_{a,\alpha}(z)|)=\left(\frac{1-|a|^2}{|1-\bar{a}z|^2}\right)^{\alpha}$$

for all $z\in D$.

The following lemma is the crucial criterion for compactness, whose proof is an easy modification of Proposition 3.11 of Ref. [1].

Lemma 2.6 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$. Then the difference $C_\varphi I_g-C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is compact if and only if when $\{f_n\}$ is a bounded sequence in B_α^ϕ with $f_n\to 0$ uniformly on compact subsets of D as $n\to\infty$, then $\|(C_\varphi I_g-C_\psi I_g)f_n\|_{\phi,\alpha}\to 0$ as $n\to\infty$. (The lemma can also apply to the operators $I_gC_\varphi-I_gC_\psi$ and $C_\varphi I_g-I_gC_\psi$)

The Schwarz-Pick type derivative $\varphi^{\#}$ of φ is defined by

$$\varphi^{\#}(z)=\frac{\mu_\alpha(z)}{\mu_\alpha(\varphi(z))}\varphi'(z).$$

Lemma 2.7 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$. Then

(i) $C_\varphi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is compact if and only if

$$\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)|<\infty$$

and

$$\lim_{|\varphi(z)|\to 1}|g(\varphi(z))\varphi^{\#}(z)|=0,$$

(ii) $I_gC_\varphi:B_\alpha^\phi\to B_\alpha^\phi$ is compact if and only if

$$\sup_{z\in D}|g(z)\varphi^{\#}(z)|<\infty$$

and

$$\lim_{|\varphi(z)|\to 1}|g(z)\varphi^{\#}(z)|=0.$$

Proof To get this lemma, we only need to change $\mu(z)$, $\mu(\varphi(z))$ in Theorem 5 of Ref. [19] to $\mu_\alpha(z)$, $\mu_\alpha(\varphi(z))$. We omit the details.

3 Boundedness and compactness of $C_\varphi I_g-C_\psi I_g$

In this section, we characterize the boundedness and compactness of $C_\varphi I_g-C_\psi I_g$ acting on B_α^ϕ. We consider the boundedness at first.

Theorem 3.1 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$. Then the following statements are equivalent:

(i) $C_\varphi I_g-C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is bounded;

(ii)

$$\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)|\rho(\varphi(z),\psi(z))<\infty \quad (1)$$

$$\sup_{z\in D}|g(\psi(z))\psi^{\#}(z)|\rho(\varphi(z),\psi(z))<\infty \quad (2)$$

$$\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)-g(\psi(z))\psi^{\#}(z)|<\infty \quad (3)$$

Proof (ii)⇒(i). Assume that (1)~(3) hold. By Remark 1, Lemma 2.2 and 2.3, for every $f\in B_\alpha^\phi$,

$$\begin{aligned}&\|(C_\varphi I_g-C_\psi I_g)f\|_{\phi,\alpha}=\\&\sup_{z\in D}\mu_\alpha(z)|\varphi'(z)f'(\varphi(z))g(\varphi(z))-\\&\psi'(z)f'(\psi(z))g(\psi(z))|=\\&\sup_{z\in D}|\mu_\alpha(\varphi(z))f'(\varphi(z))g(\varphi(z))\varphi^{\#}(z)-\\&\mu_\alpha(\psi(z))f'(\psi(z))g(\psi(z))\psi^{\#}(z)|\le\\&\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)|\cdot\\&|\mu_\alpha(\varphi(z))f'(\varphi(z))-\mu_\alpha(\psi(z))f'(\psi(z))|+\\&\sup_{z\in D}\mu_\alpha(\psi(z))|f'(\psi(z))|\cdot\\&|g(\varphi(z))\varphi^{\#}(z)-g(\psi(z))\psi^{\#}(z)|\le\\&C\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)|\cdot\\&\rho(\varphi(z),\psi(z))\|f\|_{\phi,\alpha}+\\&\sup_{z\in D}|g(\varphi(z))\varphi^{\#}(z)-\\&g(\psi(z))\psi^{\#}(z)|\,\|f\|_{\phi,\alpha}\le\\&C\|f\|_{\phi,\alpha}\end{aligned}$$

and

$$\begin{aligned}&|(C_\varphi I_g-C_\psi I_g)f(0)|=\left|\int_{\psi(0)}^{\varphi(0)}f'(\xi)g(\xi)d\xi\right|\le\\&M\left|\int_{\psi(0)}^{\varphi(0)}|f'(\xi)|d\xi\right|\le C\|f\|_{\phi,\alpha},\end{aligned}$$

where $M=\sup_{z\in K}|g(z)|$ and K is a closed subset of D containing $\varphi(0)$ and $\psi(0)$. The last inequality obtained from Remark 1 on a compact subset. This shows that $C_\varphi I_g-C_\psi I_g$ is bounded.

(i)⇒(ii). Assume that $C_\varphi I_g-C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is bounded. For every $w\in D$, setting

$$g_{w,1}(z)=\int_0^z\left|f_{\varphi(w),\alpha}(s)\frac{\psi(w)-s}{1-\overline{\psi(w)}s}\right|ds.$$

It is easy to check that $g_{w,1}\in B_\alpha^\phi$ and $\|g_{w,1}\|_{\phi,\alpha}\le 1$. Note that

$$g'_{w,1}(\varphi(w))=\frac{\rho(\varphi(w),\psi(w))}{\mu_\alpha(\varphi(w))}$$

and

$$g'_{w,1}(\psi(w))=0.$$

Therefore,

$$\begin{aligned}&C\ge\|(C_\varphi I_g-C_\psi I_g)g_{w,1}\|_{\phi,\alpha}=\\&\sup_{z\in D}\mu_\alpha(z)|\varphi'(z)g'_{w,1}(\varphi(z))g(\varphi(z))-\\&\psi'(z)g'_{w,1}(\psi(z))g(\psi(z))|\ge\\&\mu_\alpha(w)|\varphi'(w)g'_{w,1}(\varphi(w))g(\varphi(w))-\end{aligned}$$

$$\psi'(w)g'_{w,1}(\psi(w))g(\psi(w))| = |g(\varphi(w))\varphi^{\#}(w)|\rho(\varphi(w),\psi(w)).$$

For arbitrary w, we get (1).

Similarly, if we set

$$g_{w,2}(z)=\int_0^z |f_{\psi(w),\alpha}(s)|\,\mathrm{d}s,$$

then $g_{w,2}\in B_\alpha^\phi$, $\| g_{w,2}\|_{\phi,\alpha}=1$. We also get

$$\begin{aligned} C &\ge \|(C_\varphi I_g - C_\psi I_g)g_{w,2}\|_{\phi,\alpha} \ge \\ & \mu_\alpha(w)|\varphi'(w)g'_{w,2}(\varphi(w))g(\varphi(w)) - \\ & \psi'(w)g'_{w,2}(\psi(w))g(\psi(w))| \ge \\ & |g(\varphi(w))\varphi^{\#}(w) - g(\psi(w))\psi^{\#}(w)| - \\ & |g(\varphi(w))\varphi^{\#}(w)| \cdot \\ & |\mu_\alpha(\varphi(w))g'_{w,2}(\varphi(w)) - 1| = \\ & |g(\varphi(w))\varphi^{\#}(w) - g(\psi(w))\psi^{\#}(w)| - \\ & |g(\varphi(w))\varphi^{\#}(w)| \cdot \\ & |\mu_\alpha(\varphi(w))g'_{w,2}(\varphi(w)) - \\ & \mu_\alpha(\psi(w))g'_{w,2}(\psi(w))| \ge \\ & |g(\varphi(w))\varphi^{\#}(w) - g(\psi(w))\psi^{\#}(w)| - \\ & C|g(\varphi(w))\varphi^{\#}(w)| \cdot \\ & \rho(\varphi(w),\psi(w))\|g_{w,2}\|_{\phi,\alpha}. \end{aligned}$$

Analogously, (3) holds by (1).

Finally, One sees that

$$\begin{aligned} & |g(\psi(w))\psi^{\#}(w)|\rho(\varphi(w),\psi(w)) \ge \\ & |g(\varphi(w))\varphi^{\#}(w)|\rho(\varphi(w),\psi(w)) + \\ & |g(\psi(w))\psi^{\#}(w) - \\ & g(\varphi(w))\varphi^{\#}(w)|\rho(\varphi(w),\psi(w)). \end{aligned}$$

Thus (2) holds by (1) and (3). The proof is end.

Now we turn to the compactness. To discuss the compactness on B_α^ϕ, we define by $\Gamma(\varphi)$ the set of sequence $\{z_n\}$ in D such that $|\varphi(z_n)|\to 1$, we also denote by $\Gamma(\varphi)$ the set of sequence $\{z_n\}$ in D such that $|\varphi(z_n)|\to 1$ and $\varphi^{\#}(z_n)g(\varphi(z_n))\to 0$. It is clear that $\Gamma^{\#}(\varphi)\subset\Gamma(\varphi)$.

Theorem 3.2 Suppose that $0<\alpha<\infty$. Let φ, $\psi\in S(D)$ and $g\in H(D)$, $C_\varphi I_g$, $C_\psi I_g: B_\alpha^\phi\to B_\alpha^\phi$ are bounded but not compact. Then the following statements are equivalent:

(i) $C_\varphi I_g - C_\psi I_g: B_\alpha^\phi\to B_\alpha^\phi$ is compact;

(ii) Both (a) and (b) hold: (a) If $\Gamma^{\#}(\varphi)=\Gamma^{\#}(\psi)=\emptyset$, then $\Gamma^{\#}(\varphi)\subset\Gamma(\varphi)\cap\Gamma(\psi)$; (b) For $\{z_n\}\in\Gamma(\varphi)\cap\Gamma(\psi)$,

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0,$$

$$\lim_{n\to\infty}|g(\psi(z_n))\psi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0,$$

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n) - g(\psi(z_n))\psi^{\#}(z_n)|=0.$$

Proof (i)⇒(ii). By assuming that $C_\varphi I_g$ is not compact, there exists a sequence $\{z_n\}\in\Gamma^{\#}(\varphi)$ such that $|\varphi(z_n)|\to 1$ and $\varphi^{\#}(z_n)g(\varphi(z_n))\to 0$. For such sequence $\{z_n\}$, we set

$$h_{n,1}(z)=\int_0^z \left| f_{\varphi(z_n),\alpha}(s)\frac{\psi(z_n)-s}{1-\overline{\psi(z_n)}s}\right|\mathrm{d}s,$$

$$h_{n,2}(z)=\int_0^z |f_{\psi(z_n),\alpha}(s)|\mathrm{d}s.$$

It is clear that $h_{n,1}$ and $h_{n,2}$ belong to B_α^ϕ and converge to 0 uniformly on compact subset of D as $n\to\infty$. We have the following estimate

$$\begin{aligned} & \|(C_\varphi I_g - C_\psi I_g)h_{n,1}\|_{\phi,\alpha} \ge \\ & \mu_\alpha(z_n)|\varphi'(z_n)h'_{n,1}(\varphi(z_n))g(\varphi(z_n)) - \\ & \psi'(z_n)h'_{n,1}(\psi(z_n))g(\psi(z_n))| = \\ & |g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n)) \end{aligned}$$

and

$$\begin{aligned} & \|(C_\varphi I_g - C_\psi I_g)h_{n,2}\|_{\phi,\alpha} \ge \\ & \mu_\alpha(z_n)|\varphi'(z_n)h'_{n,2}(\varphi(z_n))g(\varphi(z_n)) - \\ & \psi'(z_n)h'_{n,2}(\psi(z_n))g(\psi(z_n))| \ge \\ & |g(\varphi(z_n))\varphi^{\#}(z_n) - g(\psi(z_n))\psi^{\#}(z_n)| - \\ & |g(\varphi(z_n))\varphi^{\#}(z_n)| \cdot \\ & |\mu_\alpha(\varphi(z_n))h'_{n,2}(\varphi(z_n)) - \\ & \mu_\alpha(\psi(z_n))h'_{n,2}(\psi(z_n))| \ge \\ & |g(\varphi(z_n))\varphi^{\#}(z_n) - g(\psi(z_n))\psi^{\#}(z_n)| - \\ & C|g(\varphi(z_n))\varphi^{\#}(z_n)| \cdot \\ & \rho(\varphi(z_n),\psi(z_n))\|h_{n,2}\|_{\varphi,\alpha}. \end{aligned}$$

By condition (i) and Lemma 2.6, we have

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0 \quad (4)$$

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n) - g(\psi(z_n))\psi^{\#}(z_n)|=0 \quad (5)$$

By assumption $g(\varphi(z_n))\varphi^{\#}(z_n)\not\to 0$ and (4), one sees that

$$\lim_{n\to\infty}\rho(\varphi(z_n),\psi(z_n))=0 \text{ for } \{z_n\}\in\Gamma^{\#}(\varphi) \quad (6)$$

Hence for any $\{z_n\}$ such that $|\varphi(z_n)|\to 1$, we get

$$\lim_{|\varphi(z_n)|\to 1}|g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0.$$

The same is true with the role of φ and ψ interchanged. Notice that (6) implies for $\{z_n\}\in\Gamma^{\#}(\varphi)$,

$$\lim_{|\varphi(z_n)|\to 1}|\varphi(z_n)-\psi(z_n)|=0.$$

For any sequence $\{z_n\}$ with $|\varphi(z_n)|\to 1$, $|\psi(z_n)|\to 1$ and $g(\varphi(z_n))\varphi^{\#}(z_n)\to 0$, we will use

$$\lim_{|\psi(z_n)|\to 1}|g(\psi(z_n))\psi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0$$

to obtain

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n)|=\lim_{n\to\infty}|g(\psi(z_n))\psi^{\#}(z_n)|=0.$$

Consequently we get (b).

Moreover, from (5) and (6) we can observe that if $\{z_n\}\in\Gamma^{\#}(\varphi)$, then $|\psi(z_n)|\to 1$ and $g(\psi(z_n))\psi^{\#}(z_n)\nrightarrow 0$, which means $\Gamma^{\#}(\varphi)\subset\Gamma^{\#}(\psi)$. Similarly we can obtain $\Gamma^{\#}(\psi)\subset\Gamma^{\#}(\varphi)$, which implies $\Gamma^{\#}(\varphi)=\Gamma^{\#}(\psi)$. From $\Gamma^{\#}(\varphi)\subset\Gamma(\varphi)$ and $\Gamma^{\#}(\psi)\subset\Gamma(\psi)$ we have $\Gamma^{\#}(\varphi)\subset\Gamma(\varphi)\cap\Gamma(\psi)$. Consequently we get (a).

(ii)⇒(i). To prove $C_\varphi I_g-C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is compact, we suppose it is not true. Let $\{f_n\}$ be a sequence in B_α^ϕ such that $\|f_n\|_{\phi,\alpha}\le 1$ and $f_n\to 0$ uniformly on compact subset of D. Assume that for some $\varepsilon>0$, $\|(C_\varphi I_g-C_\psi I_g)f_n\|_{\phi,\alpha}>\varepsilon$ for all n. For each n, there exists a sequence $\{z_n\}\in D$ such that

$$|g(\varphi(z_n))\varphi^{\#}(z_n)f'_n(\varphi(z_n))\mu_\alpha(\varphi(z_n))-g(\psi(z_n))\psi^{\#}(z_n)f'_n(\psi(z_n))\mu_\alpha(\psi(z_n))|>\varepsilon \quad (7)$$

which implies either $|\varphi(z_n)|\to 1$ or $|\psi(z_n)|\to 1$.

Suppose that $|\varphi(z_n)|\to 1$ and $|\psi(z_n)|\to w$. If $|w|<1$, then $\{z_n\}$ is not in $\Gamma(\varphi)\cap\Gamma(\psi)$. By condition (a), we have $g(\varphi(z_n))\varphi^{\#}(z_n)\to 0$.

On the other hand, $|\psi(z_n)|<1$ implies $f'_n(\psi(z_n))\to 0$. This contradicts (7). Thus we obtain $|w|=1$, which means $|\varphi(z_n)|\to 1$ and $|\psi(z_n)|\to 1$. By condition (b), one sees that

$$\begin{aligned}&|g(\varphi(z_n))\varphi^{\#}(z_n)f'_n(\varphi(z_n))\mu_\alpha(\varphi(z_n))-\\&\quad g(\psi(z_n))\psi^{\#}(z_n)f'_n(\psi(z_n))\mu_\alpha(\psi(z_n))|\le\\&\quad |g(\varphi(z_n))\varphi^{\#}(z_n)-g(\psi(z_n))\psi^{\#}(z_n)|\cdot\\&\quad \|f_n\|_{\phi,\alpha}+C|g(\psi(z_n))\psi^{\#}(z_n)|\cdot\\&\quad \rho(\varphi(z_n),\psi(z_n))\|f_n\|_{\phi,\alpha}.\end{aligned}$$

this contradicts (7). The proof is end.

If we remove the assumption that $C_\varphi I_g$ and $C_\psi I_g$ are not compact, we can get the next theorem.

Theorem 3.3 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$. Suppose $C_\varphi I_g$, $C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ are bounded. Then the following statements are equivalent:

(i) $C_\varphi I_g-C_\psi I_g:B_\alpha^\phi\to B_\alpha^\phi$ is compact;

(ii)

$$\lim_{|\varphi(z)|\to 1}|g(\varphi(z))\varphi^{\#}(z)|\rho(\varphi(z),\psi(z))=0 \quad (8)$$

$$\lim_{|\psi(z)|\to 1}|g(\psi(z))\psi^{\#}(z)|\rho(\varphi(z),\psi(z))=0 \quad (9)$$

$$\lim_{|\varphi(z)|\to 1,\,|\psi(z)|\to 1}|g(\varphi(z))\varphi^{\#}(z)-g(\psi(z))\psi^{\#}(z)|=0 \quad (10)$$

Proof (i)⇒(ii). Assume that both $C_\varphi I_g$ and $C_\psi I_g$ are compact. Then Lemma 2.7 implies that

$$\lim_{|\varphi(z)|\to 1}g(\varphi(z))\varphi^{\#}(z)=0,$$

$$\lim_{|\psi(z)|\to 1}g(\psi(z))\psi^{\#}(z)=0.$$

From $|\rho(\varphi(z),\psi(z))|\le 1$ we obtain (ii).

Assume that both $C_\varphi I_g$ and $C_\psi I_g$ are not compact. For any sequence $\{z_n\}$ with $|\varphi(z_n)|\to 1$, if $g(\varphi(z_n))\varphi^{\#}(z_n)\to 0$, then

$$\lim_{n\to\infty}g(\varphi(z_n))\varphi^{\#}(z_n)\rho(\varphi(z_n),\psi(z_n))=0.$$

Suppose that $\{z_n\}\in\Gamma^{\#}(\varphi)$. By Theorem 3.2, we have $\{z_n\}\in\Gamma^{\#}(\varphi)\subset\Gamma(\varphi)\cap\Gamma(\psi)$ and

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0.$$

Hence

$$\lim_{|\varphi(z_n)|\to 1}|g(\varphi(z_n))\varphi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0.$$

Similarly, we have

$$\lim_{|\psi(z_n)|\to 1}|g(\psi(z_n))\psi^{\#}(z_n)|\rho(\varphi(z_n),\psi(z_n))=0.$$

For $\{z_n\}$ such that $|\varphi(z_n)|\to 1$, $|\psi(z_n)|\to 1$, by Theorem 3.2 we can obtain

$$\lim_{n\to\infty}|g(\varphi(z_n))\varphi^{\#}(z_n)-g(\psi(z_n))\psi^{\#}(z_n)|=0.$$

Therefore, for arbitrary $\{z_n\}$, conditions (8)~(10) hold.

(ii)⇒(i). Suppose that one of the two operators $C_\varphi I_g$ and $C_\psi I_g$ is compact, for example, $C_\varphi I_g$, then from Lemma 2.7 we have

$$\lim_{|\varphi(z)|\to 1}g(\varphi(z))\varphi^{\#}(z)=0.$$

Let $\{z_n\}$ be any sequence in D such that $|\psi(z_n)|\to 1$ as $n\to\infty$. If $|\varphi(z_n)|\to 1$, from (10) we obtain

$$\lim_{n\to\infty}g(\psi(z_n))\psi^{\#}(z_n)=0.$$

Otherwise, $\rho(\varphi(z_n),\psi(z_n))\nrightarrow 0$. From (9) we get

$$\lim_{n\to\infty}g(\psi(z_n))\psi^{\#}(z_n)=0.$$

Thus we have

$$\lim_{|\psi(z)|\to 1}g(\psi(z))\psi^{\#}(z)=0.$$

Using again Lemma 2.7, we know $C_\psi I_g$ is compact, then $C_\varphi I_g-C_\psi I_g$ is compact.

If both $C_\varphi I_g$ and $C_\psi I_g$ are not compact, then

(i) follows from Theorem 3.2. The proof is end.

Similar to the proof of Theorem 3.1～3.3, we can proof the following theorem. Here we omit the details.

Theorem 3.4 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$.

(i) $I_gC_\varphi-I_gC_\psi:B_\alpha^\phi\to B_\alpha^\phi$ is bounded if and only if

$$\sup_{z\in D}|g(z)\varphi^\#(z)|\rho(\varphi(z),\psi(z))<\infty,$$
$$\sup_{z\in D}|g(z)\psi^\#(z)|\rho(\varphi(z),\psi(z))<\infty,$$
$$\sup_{z\in D}|g(z)\varphi^\#(z)-g(z)\psi^\#(z)|<\infty.$$

(ii) $I_gC_\varphi-I_gC_\psi:B_\alpha^\phi\to B_\alpha^\phi$ is compact if and only if

$$\lim_{|\varphi(z)|\to1}|g(z)\varphi^\#(z)|\rho(\varphi(z),\psi(z))=0,$$
$$\lim_{|\psi(z)|\to1}|g(z)\psi^\#(z)|\rho(\varphi(z),\psi(z))=0,$$
$$\lim_{|\varphi(z)|\to1,|\psi(z)|\to1}|g(z)\varphi^\#(z)-g(z)\psi^\#(z)|=0.$$

4 Boundedness and compactness of $C_\varphi I_g-I_gC_\varphi$

In this section, we characterize the boundedness and compactness of $C_\varphi I_g-I_gC_\psi$ acting on B_α^ϕ.

Theorem 4.1 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$. Then the following statements are equivalent:

(i) $C_\varphi I_g-I_gC_\psi:B_\alpha^\phi\to B_\alpha^\phi$ is bounded;

(ii)

$$\sup_{z\in D}|g(\varphi(z))\varphi^\#(z)|\rho(\varphi(z),\psi(z))<\infty \tag{11}$$
$$\sup_{z\in D}|g(z)\psi^\#(z)|\rho(\varphi(z),\psi(z))<\infty \tag{12}$$
$$\sup_{z\in D}|g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)|<\infty \tag{13}$$

Proof Notice that

$$\|(C_\varphi I_g-C_\psi I_g)f\|_{\phi,\alpha}=\sup_{z\in D}\mu_\alpha(z)|\varphi'(z)f'(\varphi(z))g(\varphi(z))-\psi'(z)f'(\psi(z))g(\psi(z))|,$$
$$\|(C_\varphi I_g-I_gC_\psi)f\|_{\phi,\alpha}=\sup_{z\in D}\mu_\alpha(z)|\varphi'(z)f'(\varphi(z))g(\varphi(z))-\psi'(z)f'(\psi(z))g(z)|.$$

To prove this theorem, we only need to change $g(\psi(z))$, $g(\psi(w))$ in the proof of Theorem 3.1 to $g(z)$, $g(w)$. Here we omit the details.

Theorem 4.2 Suppose that $0<\alpha<\infty$. Let $\varphi,\psi\in S(D)$ and $g\in H(D)$. Suppose $C_\varphi I_g$, $I_gC_\psi:B_\alpha^\phi\to B_\alpha^\phi$ are bounded. Then the following statements are equivalent:

(i) $C_\varphi I_g-I_gC_\psi:B_\alpha^\phi\to B_\alpha^\phi$ is compact;

(ii)

$$\lim_{|\varphi(z)|\to1}|g(\varphi(z))\varphi^\#(z)|\rho(\varphi(z),\psi(z))=0 \tag{14}$$
$$\lim_{|\psi(z)|\to1}|g(z)\psi^\#(z)|\rho(\varphi(z),\psi(z))=0 \tag{15}$$
$$\lim_{|\varphi(z)|\to1,|\psi(z)|\to1}|g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)|=0 \tag{16}$$

Proof (ii)⇒(i). Assume that $C_\varphi I_g-I_gC_\psi$ is bounded on B_α^ϕ and (14)～(16) hold. Then by Theorem 4.1, conditions (11)～(13) hold. From (14)～(16), it follows that for any $\varepsilon>0$, there exists $0<r<1$ such that

$$|g(\varphi(z))\varphi^\#(z)|\rho(\varphi(z),\psi(z))\le\varepsilon \quad \text{for } |\varphi(z)|>r \tag{17}$$
$$|g(z)\psi^\#(z)|\rho(\varphi(z),\psi(z))\le\varepsilon \quad \text{for } |\psi(z)|>r \tag{18}$$
$$|g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)|\le\varepsilon \quad \text{for } |\varphi(z)|>r,|\psi(z)|>r \tag{19}$$

Let $\{f_n\}$ be a sequence in B_α^ϕ with $\|f_n\|_{B_\alpha^\phi}\le1$ and $f_n\to0$ uniformly on compact subsets of D. By Lemma 2.6 we only need to show that $\|(C_\varphi I_g-I_gC_\psi)f_n\|_{\phi,\alpha}\to0$ as $n\to\infty$. In fact,

$$\|(C_\varphi I_g-I_gC_\psi)f_n\|_{\phi,\alpha}=\sup_{z\in D}\mu_\alpha(z)|\varphi'(z)f_n'(\varphi(z))g(\varphi(z))-\psi'(z)f_n'(\psi(z))g(z)|=$$
$$\sup_{z\in D}|\mu_\alpha(\varphi(z))f_n'(\varphi(z))g(\varphi(z))\varphi^\#(z)-\mu_\alpha(\psi(z))f_n'(\psi(z))g(z)\psi^\#(z)|=\sup_{z\in D}|I_n(z)+J_n(z)|,$$

where

$$I_n(z)=\mu_\alpha(\psi(z))f_n'(\psi(z))\cdot(g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)),$$
$$J_n(z)=g(\varphi(z))\varphi^\#(z)\cdot(\mu_\alpha(\varphi(z))f_n'(\varphi(z))-\mu_\alpha(\psi(z))f_n'(\psi(z))).$$

In what follows, we divide the argument into 4 cases.

Case 1. $|\varphi(z)|\le r$ and $|\psi(z)|\le r$. By the assumption, $\{f_n(z)\}\to0$ uniformly on $E=\{w\in D:|w|\le r\}$ as $n\to\infty$. By (13) and Cauchy's integral formula, it is easy to check that $|I_n(z)|\to0$ uniformly for all z with $|\psi(z)|\le r$ as $n\to\infty$. On the other hand, it follows from Remark 2 that

$$|\mu_\alpha(\varphi(z))f_n'(\varphi(z))-\mu_\alpha(\psi(z))f_n'(\psi(z))|\le C\rho(\varphi(z),\psi(z))\sup_{|\psi(z)|\le r}\mu_\alpha(\psi(z))|f_n'(\psi(z))|.$$

Together with (1) and the fact that $\{f_n(z)\}\to 0$ uniformly on E, we have

$$|J_n(z)|\le C|g(\varphi(z))\varphi^\#(z)|\rho(\varphi(z),\psi(z))\sup_{|w|\le r}\mu_\alpha(w)|f_n'(w)|\le C\varepsilon.$$

Case 2. $|\varphi(z)|>r$ and $|\psi(z)|\le r$. As the proof of Case 1, $|I_n(z)|\to 0$ uniformly as $n\to\infty$. On the other hand, using Lemma 2.3 and (17) we have

$$|J_n(z)|\le C\|f_n\|_{\phi,\alpha}|g(\varphi(z))\varphi^\#(z)|\cdot\rho(\varphi(z),\psi(z))\le C\varepsilon.$$

Case 3. $|\varphi(z)|>r$ and $|\psi(z)|>r$. By (19) we obtain that

$$|I_n(z)|\le\|f_n\|_{\phi,\alpha}|g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)|\le\varepsilon$$

for n sufficiently large. Meanwhile, $|J_n(z)|\to 0$ uniformly as $n\to\infty$ as the proof of Case 2.

Case 4. $|\varphi(z)|<r$ and $|\psi(z)|\ge r$. We rewrite

$$\mu_\alpha(z)|\varphi'(z)f_n'(\varphi(z))g(\varphi(z))-\psi'(z)f_n'(\psi(z))g(z)|=|P_n(z)+Q_n(z)|,$$

where

$$P_n(z)=\mu_\alpha(\varphi(z))f_n'(\varphi(z))\cdot(g(\varphi(z))\varphi^\#(z)-g(z)\psi^\#(z)),$$

$$Q_n(z)=g(z)\psi^\#(z)\cdot(\mu_\alpha(\varphi(z))f_n'(\varphi(z))-\mu_\alpha(\psi(z))f_n'(\psi(z))).$$

The desired result follows by an argument analogous to the proof of Case 2.

(i)⇒(ii). Let $\{z_n\}$ be a sequence in D such that $|\varphi(z_n)|\to 1$ as $n\to\infty$, we define the following function

$$h_{n,1}(z)=\int_0^z\left|f_{\varphi(z_n),\alpha(s)}\frac{\psi(z_n)-s}{1-\overline{\psi(z_n)}s}\right|\mathrm{d}s.$$

When $\{z_n\}$ is a sequence in D such that $|\varphi(z_n)|\to 1$, $|\psi(z_n)|\to 1$ as $n\to\infty$, we define the function

$$h_{n,2}(z)=\int_0^z|f_{\psi(z_n),\alpha}(s)|\mathrm{d}s.$$

Let $\{z_n\}$ be a sequence in D such that $|\psi(z_n)|\to 1$ as $n\to\infty$, we define the function

$$h_{n,3}(z)=\int_0^z\left|f_{\psi(z_n),\alpha(s)}\frac{\varphi(z_n)-s}{1-\overline{\varphi(z_n)}s}\right|\mathrm{d}s.$$

It is easy to check that $\{h_{n,k}:k=1,2,3\}$ converge to 0 uniformly on compact subsets of D as $n\to\infty$ and $h_{n,k}\in B_\alpha^\phi$ with $\|h_{n,k}\|_{\phi,\alpha}\le 1$ for all n. Similar to the proof of Theorem 3.2, a direct calculation shows that

$$\|(C_\varphi I_g-I_gC_\psi)h_{n,1}\|_{\phi,\alpha}\ge|g(\varphi(z_n))\varphi^\#(z_n)|\rho(\varphi(z_n),\psi(z_n)),$$

$$\|(C_\varphi I_g-I_gC_\psi)h_{n,2}\|_{\phi,\alpha}\ge|g(\varphi(z_n))\varphi^\#(z_n)-g(z_n)\psi^\#(z_n)|-C|g(\varphi(z_n))\varphi^\#(z_n)|\cdot\rho(\varphi(z_n),\psi(z_n))\|h_{n,2}\|_{\phi,\alpha},$$

$$\|(C_\varphi I_g-I_gC_\psi)h_{n,3}\|_{\phi,\alpha}\ge|g(z_n)\psi^\#(z_n)|\rho(\varphi(z_n),\psi(z_n)).$$

By the compactness of $C_\varphi I_g-I_gC_\psi$ and Lemma 2.6, (14)~(16) hold. The proof is end.

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