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一类分数阶脉冲微分方程边值问题正解的存在唯一性

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摘　要：本文利用混合单调算子的不动点定理得到了分数阶脉冲微分方程边值问题

$$\begin{cases} {}^{C}D_{0^+}^{q}u(t)=f(t,u(t),u(t)),t\in J'=J\backslash\{t_1,t_2,\cdots,t_m\},J=[0,1],\\ \Delta u(t_k)=I_k(u(t_k),u(t_k)),\Delta u'(t_k)=J_k(u(t_k),u(t_k)),k=1,2,\cdots,m,\\ au(0)-bu(1)=0,au'(0)-bu'(1)=0\end{cases}$$

存在唯一正解的新判据，其中 $1<q<2$，${}^{C}D_{0^+}^{q}$ 为 Caputo 分数阶导数.

关键词：分数阶脉冲微分方程；边值问题；正解；存在唯一性
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Existence and uniqueness of positive solutions for a class of fractional limpulsive differential equations with boundary value problems

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Abstract: By using the fixed point theorem for mixed monotone operator, a new criterion for the existence and uniqueness of positive solution of the boundary alue problems of a class of fractional impulsive differential equations

$$\begin{cases} {}^{C}D_{0^+}^{q}u(t)=f(t,u(t),u(t)),t\in J'=J\backslash\{t_1,t_2,\cdots,t_m\},J=[0,1],\\ \Delta u(t_k)=I_k(u(t_k),u(t_k)),\Delta u'(t_k)=J_k(u(t_k),u(t_k)),k=1,2,\cdots,m,\\ au(0)-bu(1)=0,au'(0)-bu'(1)=0\end{cases}$$

is established, where $1<q<2$, ${}^{C}D_{0^+}^{q}$ is the Caputo fractional derivative.

Keywords: Fractional impulsive differential equation; Boundary value problem; Positive solution; Existence and uniqueness
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1 Introduction

Recently, boundary value problems (BVPs for short) of fractional differential equations have been investigated extensively[1-10]. In most of the papers, the results demand that the nonlinear term and the impulse functions are bounded or satisfy Lipschitz conditions. For example, Zhao *et al.*[1] studied the positive solutions for the following problem:

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$$\begin{cases} {}^{C}D_{0^+}^{q} u(t) = f(t, u(t)), t \in J' = J \setminus \{t_1, t_2, \cdots, t_m\}, J = [0,1], \\ \Delta u(t_k) = I_k(u(t_k)), \Delta u'(t_k) = J_k(u(t_k)), k = 1, 2, \cdots, m, \\ au(0) - bu(1) = 0, au'(0) - bu'(1) = 0, \end{cases}$$

where $1 < q < 2$, ${}^{C}D_{0^+}^{q}$ is the Caputo fractional derivative. The results demand that the nonlinear term and the impulse functions satisfy Lipschitz conditions. Clearly, these conditions are very strong.

Inspired by the above literatures, by using fixed point theorem for mixed monotone operator, we studied the existence and uniqueness of positive solution for the following problem:

$$\begin{cases} {}^{C}D_{0^+}^{q} u(t) = f(t, u(t), u(t)), t \in J' = J \setminus \{t_1, t_2, \cdots, t_m\}, J = [0,1], \\ \Delta u(t_k) = I_k(u(t_k), u(t_k)), \Delta u'(t_k) = J_k(u(t_k), u(t_k)), k = 1, 2, \cdots, m, \\ au(0) - bu(1) = 0, au'(0) - bu'(1) = 0 \end{cases} \tag{1}$$

where ${}^{C}D_{0^+}^{q}$ is the Caputo fractional derivative, $1 < q < 2$, $f: J \times \mathbf{R}^+ \times \mathbf{R}^+ \to \mathbf{R}^+$ is jointly continuous, $I_k, J_k \in \mathbf{C}(\mathbf{R}^+ \times \mathbf{R}^+, \mathbf{R}^+)$. The impulsive points set $\{t_k\}_{k=1}^{m}$ satisfies $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$ with $u(t_k^+) = \lim_{h \to 0^+} u(t_k + h)$, $u(t_k^-) = \lim_{h \to 0^-} u(t_k + h), k = 1, 2, \cdots, m$. Our work presents some new features and improves the results of Refs. [1～8] (see Remark 2).

2 Preliminaries

Let $(E, \|\cdot\|)$ be a real Banach space which is partially ordered by a cone $P \subset E$, i.e., $x \leqslant y$ if and only if $y - x \in P$. By θ we denote the zero element of E. Recall that a non-empty closed convex set $P \subset E$ is a cone if it satisfies (i) $x \in P, r \geqslant 0 \Rightarrow rx \in P$; (ii) $x \in P, -x \in P \Rightarrow x = \theta$. In our consideration, we shall consider the Banach space $E = \{u(t): u(t) \in C(J)\}$ with the norm $\|\cdot\|$ defined by $\|u\| = \sup_{t \in J} |u(t)|$. $PC(J) = \{u \in E \mid u: J \to \mathbf{R}^+, u \in C(J'), u(t_k^+), u(t_k^-)$ exist with $u(t_k^-) = u(t_k), 1 \leqslant k \leqslant m\}$. $P = \{u \in PC(J): u(t) \geqslant 0, t \in J\}$. Obviously, $PC(J) \subset E$ is a Banach space with the norm $\|u\| = \sup_{t \in J} |u(t)|$, $P \subset PC(J)$ is a normal cone. For $w > \theta$, the set $\{u \in P: u \sim w\}$ is denoted by P_w, where $\sim$ is an equivalence relation, see Ref. [15] for details.

Definition 2.1[11,12] The fractional integral of order $q > 0$ for a function $f: [0, +\infty) \to \mathbf{R}$ is defined as

$$I_{0^+}^{q} f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-s)^{q-1} f(s) \mathrm{d}s, t > 0.$$

where $\Gamma(\cdot)$ is the gamma function.

Definition 2.2 (Generalization of classical Caputo fractional derivative)[11,12] The Caputo fractional derivative of order $q > 0$ for a function $f: [0, +\infty) \to \mathbf{R}$ is defined as

$${}^{C}D_{0+}^{q} f(t) = \frac{1}{\Gamma(n-q)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_0^t \frac{f(s) - \sum_{k=0}^{n-1} \frac{s^k}{k!} f^{(k)}(0)}{(t-s)^{q-n+1}} \mathrm{d}s, t > 0, n-1 < q < n.$$

Definition 2.3[13,14] $A: P \times P \to P$ is said to be a mixed monotone operator if $A(x, y)$ is increasing in x and decreasing in y. Element $x \in P$ is called a fixed point of A if $A(x, x) = x$.

Lemma 2.4[15] Let P be a normal cone in a real Banach space E, $A: P \times P \to P$ is a mixed monotone operator and satisfies

(A1) There exists $w \in P$ with $w \neq \theta$ such that $A(w, w) \in P_w$;

(A2) For any $u, v \in P$ and $t \in (0,1)$, there exists $\varphi(t)\in(t,1]$ such that $A(tu,t^{-1}v)\geqslant\varphi(t)A(u,v)$.

Then the operator equation $A(x,x)=x$ has unique positive solution x^* in P_w. Moreover, for any initial values $x_0, y_0\in P_w$, constructing successively the sequences

$$x_n=A(x_{n-1},y_{n-1}),\ y_n=A(y_{n-1},x_{n-1}),\quad n=1,2,\cdots,$$

we have $x_n\to x^*$, $y_n\to x^*$ as $n\to\infty$.

3 Main results

From Lemma 2.5 and Lemma 2.6 in Ref. [1], we can easily obtain the following lemmas.

Lemma 3.1 Given $h(t)\in C(J,\mathbf{R}^+)$, $1<q<2$, the unique solution of

$$\begin{cases} {}^{C}D_{0^+}^{q}u(t)=h(t),t\in J'=J\backslash\{t_1,t_2,\cdots,t_m\},J=[0,1],\\ \Delta u(t_k)=I_k(u(t_k),u(t_k)),\Delta u'(t_k)=J_k(u(t_k),u(t_k)),k=1,2,\cdots,m,\\ au(0)-bu(1)=0,au'(0)-bu'(1)=0,a>b>0\end{cases}\tag{2}$$

is formulated by

$$u(t)=\int_0^1 G_1(t,s)h(s)\mathrm{d}s+\sum_{i=1}^{m}G_2(t,t_i)J_i(u(t_i),u(t_i))+\sum_{i=1}^{m}G_3(t,t_i)I_i(u(t_i),u(t_i)),t\in J,$$

where

$$G_1(t,s)=\begin{cases}\dfrac{(t-s)^{q-1}}{\Gamma(q)}+\dfrac{b\,(1-s)^{q-1}}{(a-b)\Gamma(q)}+\dfrac{[(ab-b^2)t+b^2](1-s)^{q-2}}{(a-b)^2\Gamma(q-1)},0\leqslant s\leqslant t\leqslant 1,\\ \dfrac{b\,(1-s)^{q-1}}{(a-b)\Gamma(q)}+\dfrac{[(ab-b^2)t+b^2](1-s)^{q-2}}{(a-b)^2\Gamma(q-1)},0\leqslant t\leqslant s\leqslant 1,\end{cases}$$

$$G_2(t,t_i)=\begin{cases}\dfrac{ab}{(a-b)^2}+\dfrac{a(t-t_i)}{(a-b)},0\leqslant t_i<t\leqslant 1,i=1,2,\cdots,m,\\ \dfrac{ab}{(a-b)^2}+\dfrac{b(t-t_i)}{(a-b)},0\leqslant t<t_i\leqslant 1,i=1,2,\cdots,m,\end{cases}$$

$$G_3(t,t_i)=\begin{cases}\dfrac{a}{a-b},0\leqslant t_i<t\leqslant 1,i=1,2,\cdots,m,\\ \dfrac{b}{a-b},0\leqslant t<t_i\leqslant 1,i=1,2,\cdots,m.\end{cases}$$

***Remark* 1** When $I_k(u(t_k),u(t_k))\equiv 0$ and $J_k(u(t_k),u(t_k))\equiv 0$, $k=1,2,\cdots,m$, given $h(t)\in C(J,\mathbf{R}^+)$, $1<q<2$, the uniqe solution of the problem (2) is formulated by

$$u(t)=\int_0^1 G_1(t,s)h(s)ds,t\in J,$$

where

$$G_1(t,s)=\begin{cases}\dfrac{(t-s)^{q-1}}{\Gamma(q)}+\dfrac{b\,(1-s)^{q-1}}{(a-b)\Gamma(q)}+\dfrac{[(ab-b^2)t+b^2](1-s)^{q-2}}{(a-b)^2\Gamma(q-1)},0\leqslant s\leqslant t\leqslant 1,\\ \dfrac{b\,(1-s)^{q-1}}{(a-b)\Gamma(q)}+\dfrac{[(ab-b^2)t+b^2](1-s)^{q-2}}{(a-b)^2\Gamma(q-1)},0\leqslant t\leqslant s\leqslant 1.\end{cases}$$

Lemma 3.2 Let $a>b>0$. then Green functions $G_1(t,s)$, $G_2(t,t_i)$ and $G_3(t,t_i)$ satisfy:

(i) $G_1(t,s),G_2(t,t_i),G_3(t,t_i)\in C(J\times J,\mathbf{R}^+)$ and $G_1(t,s),G_2(t,t_i),G_3(t,t_i)>0,\forall t,t_i,s\in(0,1)$;

(ii) $\dfrac{b^2}{(a-b)^2}\leqslant G_2(t,t_i)\leqslant\dfrac{a^2}{(a-b)^2},\forall t,t_i\in J$,

$$\frac{b}{a-b}\leqslant G_3(t,t_i)\leqslant\frac{a}{a-b},\forall t,t_i\in J.$$

Theorem 3.3 Assume that

(B1) $a>b>0$, $f(t,u,v)$ is increasing in $u\in[0,+\infty)$ for fixed $t\in(0,1)$, $v\in[0,+\infty)$, and decreasing in $v\in[0,+\infty)$ for fixed $t\in(0,1)$, $u\in$

$[0,+\infty)$. $f(t,c_1,c_2)>0$ with $c_1=\min_{t\in[0,1]}w(t)$, $c_2=\max_{t\in[0,1]}w(t)$, where $w(t)=\int_0^1 G_1(t,s)\mathrm{d}s>0, t\in[0,1]$;

(B2) For all $\gamma\in(0,1), t\in(0,1), u,v\in[0,+\infty)$, there exists $\varphi_1(\gamma)\in(\gamma,1)$ such that $f(t,\gamma u,\gamma^{-1}v)\geqslant\varphi_1(\gamma)f(t,u,v)$;

(B3) $I_k(u,v), J_k(u,v)(k=1,2,\cdots,m)$ are increasing in $u\in[0,+\infty)$ for fixed $v\in[0,+\infty)$, decreasing in $v\in[0,+\infty)$ for fixed $u\in[0,+\infty)$;

(B4) For all $\gamma\in(0,1), t\in(0,1), u,v\in[0,+\infty)$, there exist $\varphi_2(\gamma),\varphi_3(\gamma)\in(\gamma,1)$ such that $I_k(\gamma u,\gamma^{-1}v)\geqslant\varphi_2(\gamma)I_k(u,v), J_k(\gamma u,\gamma^{-1}v)\geqslant\varphi_3(\gamma)J_k(u,v)$.

Then Problem (1) has unique positive solution u^* in P_w. Moreover, for any initial values $u_0,v_0\in P_w$, constructing successively the sequences

$$u_n(t)=\int_0^1 G_1(t,s)f(s,u_{n-1}(s),v_{n-1}(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(u_{n-1}(t_i),v_{n-1}(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u_{n-1}(t_i),v_{n-1}(t_i)),$$

$$v_n(t)=\int_0^1 G_1(t,s)f(s,v_{n-1}(s),u_{n-1}(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(v_{n-1}(t_i),u_{n-1}(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(v_{n-1}(t_i),u_{n-1}(t_i)), n=1,2,\cdots,$$

we have $u_n(t)\to u^*(t), v_n(t)\to u^*(t)$ as $n\to\infty$.

Proof　From Lemma 3.1, Problem (1) has an integral formulation given by

$$u(t)=\int_0^1 G_1(t,s)f(s,u(s),u(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(u(t_i),u(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u(t_i),u(t_i)), t\in J.$$

Define operator $A:P\times P\to E$ by

$$A(u,v)(t)=\int_0^1 G_1(t,s)f(s,u(s),v(s))ds+\sum_{i=1}^m G_2(t,t_i)J_i(u(t_i),v(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u(t_i),v(t_i)), t\in J.$$

From Lemma 3.2, we know that $A:P\times P\to P$.

Firstly, we prove that A is a mixed monotone operator. For $u_i,v_i(i=1,2)\in P, u_1\leqslant u_2, v_1\geqslant v_2$, from (B1), (B3) we obtain

$$\begin{aligned}A(u_1,v_1)(t)=&\int_0^1 G_1(t,s)f(s,u_1(s),v_1(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(u_1(t_i),v_1(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u_1(t_i),v_1(t_i))\leqslant\\&\int_0^1 G_1(t,s)f(s,u_2(s),v_2(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(u_2(t_i),v_2(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u_2(t_i),v_2(t_i))=\\&A(u_2,v_2)(t).\end{aligned}$$

That is to say, $A(u_1,v_1)\leqslant A(u_2,v_2)$. Now we show that A satisfies (A1). Let

$$r_1=\min_{t\in[0,1]}f(t,c_1,c_2),$$
$$r_2=\max_{t\in[0,1]}f(t,c_2,c_1).$$

From (B1), (B3) and Lemma 3.2, we get

$$\begin{aligned}A(w,w)(t)=&\int_0^1 G_1(t,s)f(s,w(s),w(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(w(t_i),w(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(w(t_i),w(t_i))\geqslant\\&\int_0^1 G_1(t,s)f(s,c_1,c_2)\mathrm{d}s\geqslant r_1\int_0^1 G_1(t,s)ds=r_1w(t)\end{aligned}$$

and

$$A(w,w)(t)=\int_0^1 G_1(t,s)f(s,w(s),w(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(w(t_i),w(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(w(t_i),w(t_i))\leqslant$$

$$\int_0^1 G_1(t,s)f(s,c_2,c_1)\mathrm{d}s+\Big(\sum_{i=1}^m \frac{a^2}{(a-b)^2}J_i(c_2,c_1)+\sum_{i=1}^m \frac{a}{a-b}I_i(c_2,c_1)\Big)\leqslant$$
$$r_2\int_0^1 G_1(t,s)\mathrm{d}s+\frac{1}{c_1}\Big(\sum_{i=1}^m \frac{a^2}{(a-b)^2}J_i(c_2,c_1)+\sum_{i=1}^m \frac{a}{a-b}I_i(c_2,c_1)\Big)w(t)=$$
$$\Big[r_2+\frac{1}{c_1}\Big(\sum_{i=1}^m \frac{a^2}{(a-b)^2}J_i(c_2,c_1)+\sum_{i=1}^m \frac{a}{a-b}I_i(c_2,c_1)\Big)\Big]w(t).$$

That is to say,

$$r_1 w(t)\leqslant A(w,w)(t)\leqslant\Big[r_2+\frac{1}{c_1}\Big(\sum_{i=1}^m \frac{a^2}{(a-b)^2}J_i(c_2,c_1)+\sum_{i=1}^m \frac{a}{a-b}I_i(c_2,c_1)\Big)\Big]w(t).$$

From (B1), we have $r_1=\min\limits_{t\in[0,1]} f(t,c_1,c_2)>0$ and

$$r_2+\frac{1}{c_1}\Big(\sum_{i=1}^m \frac{a^2}{(a-b)^2}J_i(c_2,c_1)+\sum_{i=1}^m \frac{a}{a-b}I_i(c_2,c_1)\Big)>r_1>0.$$

So we prove that $A(w,w)\in P_w$.

Next we show that A satisfies (A2). For any $u,v\in P$ and $\gamma\in(0,1)$, let

$$\varphi(\gamma)=\min\{\varphi_1(\gamma),\varphi_2(\gamma),\varphi_3(\gamma)\}.$$

Then $\varphi(\gamma)\in(\gamma,1)$. From (B2) and (B4), we know that

$$A(\gamma u,\gamma^{-1}v)(t)=\int_0^1 G_1(t,s)f(s,\gamma u(s),\gamma^{-1}v(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(\gamma u(t_i),\gamma^{-1}v(t_i))+$$
$$\sum_{i=1}^m G_3(t,t_i)I_i(\gamma u(t_i),\gamma^{-1}v(t_i))\geqslant\varphi_1(\gamma)\int_0^1 G_1(t,s)f(s,u(s),v(s))\mathrm{d}s+$$
$$\varphi_2(\gamma)\sum_{i=1}^m G_2(t,t_i)J_i(u(t_i),v(t_i))+\varphi_3(\gamma)\sum_{i=1}^m G_3(t,t_i)I_i(u(t_i),v(t_i))\geqslant$$
$$\varphi(\gamma)\Big(\int_0^1 G_1(t,s)f(s,u(s),v(s))\mathrm{d}s+\sum_{i=1}^m G_2(t,t_i)J_i(u(t_i),v(t_i))+\sum_{i=1}^m G_3(t,t_i)I_i(u(t_i),v(t_i))\Big)=$$
$$\varphi(\gamma)A(u,v)(t).$$

Thus $A(\gamma u,\gamma^{-1}v)\geqslant\varphi(\gamma)A(u,v)$ for $u,v\in P$ and $\gamma\in(0,1)$.

Finally, by using Lemma 2.4, the result holds.

From Theorem 3.3 and Remark 1, we can easily obtain the following corollary.

Corollary 3.4 Suppose that $I_k(u(t_k),u(t_k))\equiv 0$ and $J_k(u(t_k),u(t_k))\equiv 0, k=1,2,\cdots,m$. Assume that (B1) and (B2) hold. Then Problem (1) has unique positive solution u^* in P_w. Moreover, for any initial values $u_0,v_0\in P_w$, constructing successively the sequences

$$u_n(t)=\int_0^1 G_1(t,s)f(s,u_{n-1}(s),v_{n-1}(s))\mathrm{d}s,$$
$$v_n(t)=\int_0^1 G_1(t,s)f(s,v_{n-1}(s),u_{n-1}(s))\mathrm{d}s, n=1,2,\cdots,$$

we have $u_n(t)\to u^*(t), v_n(t)\to u^*(t)$ as $n\to\infty$.

Remark 2 Comparing the main results in Refs.[1～8], our new results don't demand that the nonlinear term and the impulse functions are bounded or satisfy Lipschitz conditions. Thus we improve the corresponding results of predecessors to some degree.

4 Illustrative example

Example 4.1 Consider the following BVPs:

$$\begin{cases} {}^{C}D_{0^+}^{\frac{3}{2}}u(t) = (u(t))^{\frac{1}{3}} + (u(t))^{-\frac{1}{3}}, t \in [0,1], t \neq \frac{1}{2}, \\ \Delta u(\frac{1}{2}) = (u(\frac{1}{2}))^{\frac{1}{4}} + (u(\frac{1}{2}))^{-\frac{1}{4}}, \Delta u'(\frac{1}{2}) = (u(\frac{1}{2}))^{\frac{1}{5}} + (u(\frac{1}{2}))^{-\frac{1}{5}}, \\ 2u(0) - u(1) = 0, 2u'(0) - u'(1) = 0 \end{cases} \tag{3}$$

In this case, $q=\frac{3}{2}, t_1=\frac{1}{2}, a=2, b=1$,

$$f(t,u(t),u(t))=(u(t))^{\frac{1}{3}}+(u(t))^{-\frac{1}{3}},$$

$$I_1(u(t_1),u(t_1))=(u(\frac{1}{2}))^{\frac{1}{4}}+(u(\frac{1}{2}))^{-\frac{1}{4}},$$

$$J_1(u(t_1),u(t_1))=(u(\frac{1}{2}))^{\frac{1}{5}}+(u(\frac{1}{2}))^{-\frac{1}{5}},$$

By a simple computation we have

$$G_1(t,s)=\begin{cases} \frac{(t-s)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\frac{(1-s)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\frac{\frac{1}{2}t(1-s)^{-\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\frac{\frac{1}{2}(1-s)^{-\frac{1}{2}}}{\Gamma(\frac{3}{2})}, 0\leqslant s\leqslant t\leqslant 1, \\ \frac{(1-s)^{\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\frac{\frac{1}{2}t(1-s)^{-\frac{1}{2}}}{\Gamma(\frac{3}{2})}+\frac{\frac{1}{2}(1-s)^{-\frac{1}{2}}}{\Gamma(\frac{3}{2})}, 0\leqslant t\leqslant s\leqslant 1, \end{cases}$$

$$w(t) = \int_0^1 G_1(t,s)ds = \frac{1}{\Gamma(\frac{3}{2})}(\frac{2}{3}t^{\frac{3}{2}} + t + \frac{5}{3}),$$

$$c_1 = \min_{t\in[0,1]} w(t) = \frac{5}{3\Gamma(\frac{3}{2})},$$

$$c_2 = \max_{t\in[0,1]} w(t) = \frac{10}{3\Gamma(\frac{3}{2})}.$$

Then $f(t,c_1,c_2)>0$. From the expressions of $f(t,u(t),u(t)), I_1(u(t_1),u(t_1)), J_1(u(t_1),u(t_1))$, it is obvious that (B1) and (B3) hold.

Moreover, for any $\gamma\in(0,1), t\in(0,1), u,v \in[0,+\infty)$, we get

$$f(t,\gamma u,\gamma^{-1}v)=\gamma^{\frac{1}{3}}u^{\frac{1}{3}}+\gamma^{\frac{1}{3}}v^{-\frac{1}{3}}\geqslant \gamma^{\frac{5}{12}}(u^{\frac{1}{3}}+v^{-\frac{1}{3}})=\varphi_1(\gamma)f(t,u,v),$$

$$I_1(\gamma u,\gamma^{-1}v)=\gamma^{\frac{1}{4}}u^{\frac{1}{4}}+\gamma^{\frac{1}{4}}v^{-\frac{1}{4}}\geqslant \gamma^{\frac{1}{3}}(u^{\frac{1}{4}}+v^{-\frac{1}{4}})=\varphi_2(\gamma)I_k(u,v),$$

$$J_1(\gamma u,\gamma^{-1}v)=\gamma^{\frac{1}{5}}u^{\frac{1}{5}}+\gamma^{\frac{1}{5}}v^{-\frac{1}{5}}\geqslant \gamma^{\frac{4}{15}}(u^{\frac{1}{5}}+v^{-\frac{1}{5}})=\varphi_3(\gamma)J_k(u,v).$$

Thus (B2) and (B4) hold. By Theorem 3.3, we know that the BVPs (3) has an unique positive solution in P_w.

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