doi：10.3969/j.issn.0490-6756.2020.06.007

关于单位分数的 Lazar 问题

卢　健[1]，李　懋[2]，邱　敏[3]
(1. 四川大学数学学院，成都 610064；2. 西南大学数学与统计学院，重庆 400715；
3. 西华大学理学院，成都 610039)

摘　要：设 n 为任意正整数. Erdös-Straus 猜想是指当 $n\geqslant 2$ 时，Diophantine 方程 $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ 总有正整数解 (x,y,z). 设 $p\geqslant 5$ 为任意素数. 最近，Lazar 证明 Diophantine 方程 $\frac{4}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ 在区域 $xy<\sqrt{z/2}$ 内没有 x 与 y 互素的正整数解 (x,y,z). 同时，Lazar 提出问题：在上述方程中以 $5/p$ 替换 $4/p$，是否有类似结果？这也是 Sierpinski 提出的一个猜想. 本文证明 Diophantine 方程 $\frac{a}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ 没有满足 x,y 互素且 $xy<\sqrt{z/2}$ 的正整数解 (x,y,z)，其中 a 为满足 $a<7\leqslant p$ 的正整数. 这回答了上述 Lazar 问题，推广了 Lazar 的结果. 证明方法和工具主要是利用有理数 $\frac{a}{p}$ 的连分数表示.

关键词：Diophantine 方程；连分数；渐近分数；Erdös-Straus 猜想
中图分类号：O156.1；O156.7　　**文献标识码**：A　　**文章编号**：0490-6756(2020)06-1067-06

On a problem of Lazar on unit fractions

LU Jian[1], *LI Mao*[2], *QIU Min*[3]
(1. School of Mathematics, Sichuan University, Chengdu 610064, China;
2. School of Mathematics and Statistics, Southwest University, Chongqing 400715, China;
3. School of Science, Xihua University, Chengdu 610039, China)

Abstract: Let n be a positive integer. The well-known Erdös-Straus conjecture asserts that the positive integral solution of the Diophantine equation $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ always exists when $n\geqslant 2$. Recently, Lazar investigated some properties of the solutions to the above Diophantine equation in the special case that n is a prime number. Let $p\geqslant 5$ be a prime number. Lazar showed that there are no triple of positive integers (x,y,z) which is solution of the Diophantine equation $\frac{4}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ in the range $xy<\sqrt{z/2}$ and $(x,y)=1$. Meanwhile, Lazar pointed out that it would be interesting to find an analog of this result for $5/p$ instead of $4/p$, which is also a conjecture due to Sierpinski. In this paper, we answer Lazar's question affirmatively and also extended Lazar's result by showing that the Diophantine equation $\frac{a}{p}=\frac{1}{x}+\frac{1}{y}+$

收稿日期：2020-03-26
基金项目：国家自然科学基金 (11771304)
作者简介：卢健 (1995－)，男，云南文山人，硕士研究生，主要研究方向为数论. E-mail：1797039012@qq.com
通讯作者：邱敏. E-mail：qiumin126@126.com

$\frac{1}{z}$ does not have any integer solution (x,y,z) such that x and y are coprime and $xy<\sqrt{z/2}$, where a is a positive integer such that $a<7\leqslant p$. Our proof mainly uses the continued fraction expansion of $\frac{a}{p}$.

Keywords: Diophantine equation; Continued fraction; Convergent; Erdös-Straus conjecture
(2010 MSC 11L05)

1 Introduction

The Rhind papyrus is amongst the oldest written mathematics that has come down to us, which concerns the representation of rational number as the sum of unit fractions:

$$\frac{m}{n}=\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_k}.$$

This has suggested numerous problems, many of which are unsolved, and continues to suggest new problems. One of these problems is the Erdös-Straus conjecture[1-17], which concerns the following Diophantine equation

$$\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \tag{1}$$

Let $\mathbf{N}$ denote the set of all the positive integers. For any positive integer n, let $f(n)$ denote the number of positive integral solutions $(x,y,z)\in\mathbf{N}^3$ to the Diophantine equation (1), that is,

$$f(n):=\#\{(x,y,z):\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z},\ (x,y,z)\in\mathbf{N}^3\}.$$

Then one has

$$f(1)=0,f(2)=3,f(3)=12,f(4)=10,\ f(5)=12,\cdots.$$

The Erdös-Straus conjecture asserts that $f(n)>0$ for all $n\geqslant 2$ (see, for example, [3,5]). Some related results are provided in Refs. [1,4,6-7,9-11,13].

In 2015, Elsholtz and Tao[2] obtained a number of upper and lower bounds of $f(n)$ or $f(p)$ for typical values of natural number n and prime p. Recently, Lazar[12] assumed that n is an odd prime number p and showed that $\frac{4}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ cannot have integeral solution such that x and y are coprime with $xy<\sqrt{z/2}$. At the end of Ref. [12], Lazar pointed out that it would be interesting to find an analog of the main result for $\frac{a}{p}$ with $a>4$ instead of $\frac{4}{p}$. So, in this paper, we present an analog of this result for the case $a<7$. That is, we have the following main result of this paper.

Theorem 1.1 Let $p\geqslant 7$ be an arbitrary prime number and a be an integer with $1\leqslant a\leqslant 6$. Then there are no triple (x,y,z) of positive integers which is a solution of the Diophantine equation

$$\frac{a}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \tag{2}$$

in the range $xy<\sqrt{z/2}$ with $\gcd(x,y)=1$.

Evidently, Theorem 1.1 gives an affirmative answer to Lazar's question[12].

2 Several lemmas

For any positive integer n, let $a_0,a_1,\cdots,a_n$ be real numbers with $a_i>0$ for $i=1,\cdots,n$. We define the finite simple continued fraction, denoted by $[a_0;a_1,\cdots,a_n]$, as follows:

$$[a_0;a_1,\cdots,a_n]:=a_0+\cfrac{1}{a_1+\cfrac{1}{a_2+\cfrac{1}{\ddots\cfrac{1}{a_{n-1}+\cfrac{1}{a_n}}}}}.$$

The k-th convergents of the continued fraction $[a_0;a_1,\cdots,a_n]$ is defined by $[a_0;a_1,\cdots,a_k]$, where k is a non-negative integer with $k\leqslant n$.

Before giving the proof of Theorem 1.1, we need the following classical result of the theory of continued fraction which can be found in Ref. [8].

Lemma 2.1[8] Let m,n,r and s be positive integers with $\gcd(r,s)=1$. If

$$\left|\frac{m}{n}-\frac{r}{s}\right|<\frac{1}{2s^2},$$

then $\frac{r}{s}$ is one of the convergents of $\frac{m}{n}$.

In the rest of this paper, we always let $p\geqslant 7$ be a prime number. The following results are about the continued fraction expansions of $\frac{3}{p}$, $\frac{5}{p}$ and $\frac{6}{p}$.

Lemma 2.2　Let $p\geqslant 7$ be a prime number. Then each of the following is true:

(a) If $p\equiv 1 \pmod 3$, then $\frac{3}{p}=[0;\frac{p-1}{3},3]$ and the convergents are $\{0,\frac{3}{p-1},\frac{3}{p}\}$;

(b) If $p\equiv 2 \pmod 3$, then $\frac{3}{p}=[0;\frac{p-2}{3},1,2]$ and the convergents are $\{0,\frac{3}{p-2},\frac{3}{p+1},\frac{3}{p}\}$.

Proof　The continued fraction of $\frac{3}{p}$ is entirely determined by the Euclidean algorithm between 3 and p.

(a) Let $p\equiv 1 \pmod 3$. Then by the Euclidean algorithm, we obtain that

$$3=p\times 0+3,\ p=3\times\frac{p-1}{3}+1,\ 3=1\times 3+0.$$

Thus $\frac{3}{p}=[0;\frac{p-1}{3},3]$. The successive convergents are given by

$$c_0=0,\ c_1=[0;\frac{p-1}{3}]=\frac{1}{\frac{p-1}{3}}=\frac{3}{p-1},$$

$$c_2=[0;\frac{p-1}{3},3]=\frac{3}{p}.$$

So the convergents are $\{0,\frac{3}{p-1},\frac{3}{p}\}$.

(b) Let $p\equiv 2 \pmod 3$. Then we deduce that

$$3=p\times 0+3,\ p=3\times\frac{p-2}{3}+2,\ 3=2\times 1+1,$$
$$2=1\times 2+0.$$

It follows that $\frac{3}{p}=[0;\frac{p-2}{3},1,2]$. The successive convergents are given by

$$c_0=0,\ c_1=[0;\frac{p-2}{3}]=\frac{1}{\frac{p-2}{3}}=\frac{3}{p-2},$$

$$c_2=[0;\frac{p-2}{3},1]=\frac{1}{\frac{p-2}{3}+1}=\frac{3}{p+1}$$

and

$$c_3=[0;\frac{p-2}{3},1,2]=\frac{3}{p}.$$

So the convergents are $\{0,\frac{3}{p-2},\frac{3}{p+1},\frac{3}{p}\}$ as expected. This completes the proof of Lemma 2.2.

Similarly, we can get the continued fraction expansion of $\frac{5}{p}$ as follows.

Lemma 2.3　Let $p\geqslant 7$ be a prime number. Then each of the following is true:

(a) If $p\equiv 1 \pmod 5$, then $\frac{5}{p}=[0;\frac{p-1}{5},5]$ and the convergents are $\{0,\frac{5}{p-1},\frac{5}{p}\}$;

(b) If $p\equiv 2 \pmod 5$, then $\frac{5}{p}=[0;\frac{p-2}{5},2,2]$ and the convergents are $\{0,\frac{5}{p-2},\frac{10}{2p+1},\frac{5}{p}\}$;

(c) If $p\equiv 3 \pmod 5$, then $\frac{5}{p}=[0;\frac{p-3}{5},1,1,2]$ and the convergents are $\{0,\frac{5}{p-3},\frac{5}{p+2},\frac{10}{2p-1},\frac{5}{p}\}$;

(d) If $p\equiv 4 \pmod 5$, then $\frac{5}{p}=[0;\frac{p-4}{5},1,4]$ and the convergents are $\{0,\frac{5}{p-4},\frac{5}{p+1},\frac{5}{p}\}$.

Proof　The continued fraction of $\frac{5}{p}$ is entirely determined by the Euclidean algorithm between 5 and p.

(a) If $p\equiv 1 \pmod 5$, then the Euclidean algorithm give us that

$$5=p\times 0+5,\ p=5\times\frac{p-1}{5}+1,\ 5=1\times 5+0.$$

Hence one derives that $\frac{5}{p}=[0;\frac{p-1}{5},5]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-1}{5}]=\frac{1}{\frac{p-1}{5}}=\frac{5}{p-1},$$

$$c_2=[0;\frac{p-1}{5},5]=\frac{5}{p}.$$

So the convergents are $\{0,\frac{5}{p-1},\frac{5}{p}\}$.

(b) If $p\equiv 2 \pmod 5$, then we obtain that

$$5=p\times 0+5, p=5\times\frac{p-2}{5}+2, 5=2\times 2+1,$$
$$2=1\times 2+0.$$

This infers that $\frac{5}{p}=[0;\frac{p-2}{5},2,2]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-2}{5}]=\frac{1}{\frac{p-2}{5}}=\frac{5}{p-2},$$

$$c_2=[0;\frac{p-2}{5},2]=\frac{1}{\frac{p-2}{5}+\frac{1}{2}}=\frac{10}{2p+1}$$

and

$$c_3=[0;\frac{p-2}{5},2,2]=\frac{5}{p}.$$

Hence the convergents are $\{0,\frac{5}{p-2},\frac{10}{2p+1},\frac{5}{p}\}$.

(c) If $p\equiv 3 \pmod 5$, then one gets that

$$5=p\times 0+5, p=5\times\frac{p-3}{5}+3, 5=3\times 1+2,$$
$$3=2\times 1+1, 2=1\times 2+0.$$

It follows that $\frac{5}{p}=[0;\frac{p-3}{5},1,1,2]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-3}{5}]=\frac{1}{\frac{p-3}{5}}=\frac{5}{p-3},$$

$$c_2=[0;\frac{p-3}{5},1]=\frac{1}{\frac{p-3}{5}+1}=\frac{5}{p+2},$$

$$c_3=[0;\frac{p-3}{5},1,1]=\frac{1}{\frac{p-3}{5}+\frac{1}{1+1}}=\frac{10}{2p-1}$$

and

$$c_4=[0;\frac{p-3}{5},1,1,2]=\frac{5}{p}.$$

Thus the convergents are $\{0,\frac{5}{p-3},\frac{5}{p+2},\frac{10}{2p-1},\frac{5}{p}\}$.

(d) If $p\equiv 4 \pmod 5$, then we deduce that

$$5=p\times 0+5, p=5\times\frac{p-4}{5}+4, 5=4\times 1+1,$$
$$4=1\times 4+0.$$

Hence $\frac{5}{p}=[0;\frac{p-4}{5},1,4]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-4}{5}]=\frac{1}{\frac{p-4}{5}}=\frac{5}{p-4},$$

$$c_2=[0;\frac{p-4}{5},1]=\frac{1}{\frac{p-4}{5}+1}=\frac{5}{p+1}$$

and

$$c_3=[0;\frac{p-4}{5},1,4]=\frac{5}{p}.$$

It implies that the convergents are $\{0,\frac{5}{p-4},\frac{5}{p+1},\frac{5}{p}\}$. This completes the proof of Lemma 2.3.

Finally, we present the continued fraction expansion of $\frac{6}{p}$.

Lemma 2.4 Let $p\geqslant 7$ be a prime number. Then each of the following is true:

(a) If $p\equiv 1 \pmod 6$, then $\frac{6}{p}=[0;\frac{p-1}{6},6]$ and the convergents are $\{0,\frac{6}{p-1},\frac{6}{p}\}$;

(b) If $p\equiv 5 \pmod 6$, then $\frac{6}{p}=[0;\frac{p-5}{6},1,5]$ and the convergents are $\{0,\frac{6}{p-5},\frac{6}{p+1},\frac{6}{p}\}$.

Proof The continued fraction of $\frac{6}{p}$ is entirely determined by the Euclidean algorithm between 6 and p.

(a) If $p\equiv 1 \pmod 6$, then by the Euclidean algorithm one obtains that

$$6=p\times 0+6, p=6\times\frac{p-1}{6}+1, 6=1\times 6+0.$$

Thus we get that $\frac{6}{p}=[0;\frac{p-1}{6},6]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-1}{6}]=\frac{1}{\frac{p-1}{6}}=\frac{6}{p-1},$$

$$c_2=[0;\frac{p-1}{6},6]=\frac{6}{p}.$$

Hence the convergents are $\{0,\frac{6}{p-1},\frac{6}{p}\}$.

(b) If $p\equiv 5 \pmod 6$, then one derives that

$$6=p\times 0+6, p=6\times\frac{p-5}{6}+5, 6=5\times 1+1,$$
$$5=1\times 5+0.$$

It follows that $\frac{6}{p}=[0;\frac{p-5}{6},1,5]$. The successive convergents are given by

$$c_0=0, c_1=[0;\frac{p-5}{6}]=\frac{1}{\frac{p-5}{6}}=\frac{6}{p-5},$$

$$c_2=[0;\frac{p-5}{6},1]=\frac{1}{\frac{p-5}{6}+1}=\frac{6}{p+1}$$

and

$$c_3=[0;\frac{p-5}{6},1,5]=\frac{6}{p}.$$

So the convergents are $\{0,\frac{6}{p-5},\frac{6}{p+1},\frac{6}{p}\}$. Thus Lemma 2.4 is proved.

3 The proof of Theorem 1.1

According to Lazar's result[12], Theorem 1.1 is true when $a=4$. So we just need to prove Theorem 1.1 for the remaining cases $a\in\{1,2,3,5,6\}$. This will be done in what follows.

In the following we let $a\in\{1,2,3,5,6\}$. Fix an arbitrarily large integer $z_0>0$ in the range $xy<\sqrt{z/2}$. Then we can rewrite equation (2) as

$$\frac{a}{p}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z_0},$$

which is equivalent to the following Diophantine equation

$$\left|\frac{a}{p}-\frac{x+y}{xy}\right|=\frac{1}{z_0}.$$

In what follows, we will try to solve above Diophantine equation with $(x,y)\in\mathbf{N}^2$. Since $xy<\sqrt{z_0/2}$, we derive the following inequality

$$\left|\frac{a}{p}-\frac{x+y}{xy}\right|<\frac{1}{2x^2y^2} \quad (3)$$

By Lemma 2.1 and (3), we can obtain that the rational number $\frac{x+y}{xy}$ must be one of the convergents of $\frac{a}{p}$. Assume that the continued fraction expansion of $\frac{a}{p}$ is $\frac{a}{p}=[a_0;a_1,\cdots,a_l]$. Then there exists some integer k with $1\leqslant k\leqslant l$ such that

$$\frac{x+y}{xy}=[a_0;a_1,\cdots,a_k]=\frac{p_k}{q_k}.$$

Since x and y are coprime, $x+y$ and xy are also coprime. Our fractions are reduced by deducing that $x+y=p_k$ and $xy=q_k$. The fact that such x and y may exist relies on the solvability in $\mathbf{N}$ of the following quadratic equation

$$X^2-p_kX+q_k=0.$$

That means, if equation (2) has integral solutions in the range $xy<\sqrt{z/2}$, then we must have $D_k=p_k^2-4q_k\geqslant 0$. Since $z_0\neq 0$, one gets that $\frac{p_k}{q_k}\neq\frac{a}{p}$. Then the original question turns into finding q_k and p_k. Since $a\in\{1,2,3,5,6\}$, we can divide the proof into the following five cases.

Case 1 $a=1$. Then the continued fraction of $\frac{1}{p}$ is itself, i.e., $\frac{1}{p}=[0;p]$, and the convergents are $\{0,\frac{1}{p}\}$. Then $\frac{p_k}{q_k}=0$ or $\frac{p_k}{q_k}=\frac{1}{p}$. But $\frac{p_k}{q_k}\neq 0$ and $\frac{p_k}{q_k}\neq\frac{a}{p}=\frac{1}{p}$. Hence the equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$ when $a=1$. So Theorem 1.1 is true in this case.

Case 2 $a=2$. Since p is a prime with $p\geqslant 7$, we have $p\equiv 1 \pmod 2$. By Euclidean algorithm, one deduces that

$$2=p\times 0+2, p=2\times\frac{p-1}{2}+1, 2=1\times 2+0.$$

It follows that $\frac{2}{p}=[0;\frac{p-1}{2},2]$, and the convergents are $\{0,\frac{2}{p-1},\frac{2}{p}\}$. Since $\frac{p_k}{q_k}\neq 0$ and $\frac{p_k}{q_k}\neq\frac{a}{p}$, we derive that $\frac{p_k}{q_k}=\frac{2}{p-1}=\frac{1}{(p-1)/2}$. This implies that $p_k=1$, $q_k=\frac{p-1}{2}$ and $D_k=p_k^2-4q_k=1-2(p-1)<0$. Hence equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$ when $a=2$. Thus Theorem 1.1 is proved in this case.

Case 3 $a=3$. After reducing all the fractions in Lemma 2.2, we obtain that

$$\frac{p_k}{q_k}=\begin{cases}\frac{1}{(p-1)/3}, & \text{if } p\equiv 1 \pmod 3;\\ \frac{1}{(p-2)/3} \text{ or } \frac{1}{(p+1)/3}, & \text{if } p\equiv 2 \pmod 3.\end{cases}$$

Note that the numerator of all cases is 1. Therefore $p_k=1$ and $D_k=1-4q_k<0$. Hence equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$ in this case. So Theorem 1.1 is true when $a=3$.

Case 4 $a=5$. By using Lemma 2.3, after reduced all the fractions, we deduce that

$$\frac{p_k}{q_k}=\begin{cases}\frac{1}{(p-1)/5}, \text{ if } p\equiv 1(\bmod 5);\\ \frac{1}{(p-2)/5} \text{ or } \frac{2}{(2p+1)/5}, \text{ if } p\equiv 2(\bmod 5);\\ \frac{1}{(p-3)/5} \text{ or } \frac{1}{(p+2)/5} \text{ or } \frac{2}{(2p-1)/5}, \text{ if } p\equiv 3(\bmod 5);\\ \frac{1}{(p-4)/5} \text{ or } \frac{1}{(p+1)/5}, \text{ if } p\equiv 4(\bmod 5).\end{cases}$$

It infers that the numerator of all the cases is 1 or 2. Therefore $p_k=1$ or $p_k=2$ and $D_k=1-4q_k$ or $D_k=4-4q_k$. Since p_k and q_k are positive integers, one derives that $D_k\leqslant 0$. If $D_k<0$, then it follows immediately that the equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$. If $D_k=0$, then we must have $p_k=2$ and $q_k=1$, from which one deduces that $x=y=1$. But $x=y=1$ cannot be a solution of equation (2) since $a<7\leqslant p$ and $z\geqslant 1$. Thus the equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$ when $a=5$. Hence Theorem 1.1 is proved in this case.

Case 5 $a=6$. From Lemma 2.4, after reduced all the fractions, we obtain that

$$\frac{p_k}{q_k}=\begin{cases}\frac{1}{(p-1)/6}, \text{ if } p\equiv 1(\bmod 6);\\ \frac{1}{(p-5)/6} \text{ or } \frac{1}{(p+1)/6}, \text{ if } p\equiv 5(\bmod 6).\end{cases}$$

Thus the numerator of all the cases is 1. This implies that $p_k=1$ and $D_k=1-4q_k$. Since p_k and q_k are positive integers, one gets that $D_k<0$. Hence the equation (2) has no integral solutions in the range $xy<\sqrt{z/2}$ when $a=6$. So Theorem 1.1 is proved in this case. This finishes the proof of Theorem 1.1.

References:

[1] Bai T T. On $\frac{1}{w}+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{2}$ and some of its generalizations [J]. J Inequal Appl, 2018, 197: 1.

[2] Elsholtz C, Tao T. Counting the number of solutions to the Erdös-Straus equation on unit fractions [J]. J Aust Math Soc, 2013, 94: 50.

[3] Erdös P. Az $1/x_1+1/x_2+\cdots+1/x_n=a/b$ egyenlet egesz szanu megoldasairol [J]. Mat Lapok, 1950, 1: 192.

[4] Göral H, Sertbas D. Density and finiteness results on sums of fractions [J]. Proc Amer Math Soc, 2019, 147: 567.

[5] Guy R. Unsolved problems in number theory [M]. New York: Springer-Verlag, 1994.

[6] Hu S N, Hong S F, Zhao W. The number of rational points of a family of hypersurfaces over finite fields [J]. J Number Theory, 2015, 156: 135.

[7] Jia C H. The estimate for mean values on prime numbers relative to $\frac{4}{p}=\frac{1}{n_1}+\frac{1}{n_2}+\frac{1}{n_3}$ [J]. Sci China Math, 2012, 55: 465.

[8] Khinchin A Y. Continued fractions [M]. New York: Dover, 1997.

[9] Ko C, Sun Q, Chang S J. On equations $4/n=1/x+1/y+1/z$ [J]. Acta Sci Natur Szechuanensis, 1964, 2: 21.

[10] Lapkova K. Explicit upper bound for an average number of divisors of quadratic polynomials [J]. Arch Math, 2016, 106: 247.

[11] Lapkova K. On the average number of divisors of reducible quadratic polynomials [J]. J Number Theory, 2017, 180: 710.

[12] Lazar Y. A remark on a conjecture of Erdös and Straus [EB/OL]. arXiv: 2003. 01237.

[13] Li D L. On the equation $4/n=1/x+1/y+1/z$ [J]. J Number Theory, 1981, 13: 485.

[14] Luca F, Pappalardi F. On ternary Egyptian fractions with prime denominator [J]. Res Number theory, 2019, 5: Art34.

[15] Subburam S, Togbé A. A note on the Erdös - Straus conjecture [J]. Period MathHungar, 2016, 72: 43.

[16] Webb W A. On $4/n=1/x+1/y+1/z$ [J]. Proc Amer Math Soc, 1970, 25: 578.

[17] Zhao J Y, Hong S F, Zhu C X. The number of rational points of certain quartic diagonal hypersurfaces over finite fields [J]. AIMS Math, 2020, 5: 2710.

引用本文格式:

中 文：卢健，李懋，邱敏. 关于单位分数的 Lazar 问题[J]. 四川大学学报：自然科学版，2020，57：1067.

英 文：Lu J，Li M，Qiu M. On a problem of Lazar on unit fractions [J]. J Sichuan Univ：Nat Sci Ed，2020，57：1067.