# 第一类 Cartan-Hartogs 域上的 Bers 型空间上的加权复合算子

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摘 要:设 $Y_I(N, m, n; K)$ 是第一类 Cartan-Hartogs 域, $\varphi$ 是  $Y_I(N, m, n; K)$ 上的全纯自映射, $H(Y_I(N, m, n; K))$ 是  $Y_I(N, m, n; K)$ 上的全纯函数集合, $u \in H(Y_I(N, m, n; K))$ . 本文运用  $Y_I(N, m, n; K)$ 上的广义华-矩阵不等式给出了  $Y_I(N, m, n; K)$ 上的 Bers 型空间上的加权复合算子  $W_{\sigma,u}$ 的有界性和紧致性的刻画.

关键词:加权复合算子;广义 Hua-矩阵不等式;第一类 Cartan-Hartogs 域; Bers 型空间;有界性;紧致性

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# Weighted composition operators on Bers-type spaces of the first kind Cartan-Hartogs domains

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**Abstract:** Let  $Y_I(N,m,n;K)$  be the first kind Cartan-Hartogs domain,  $\varphi$  a holomorphic self-map of  $Y_I(N,m,n;K)$  and  $u \in H(Y_I(N,m,n;K))$  the set of all holomorphic functions on  $Y_I(N,m,n;K)$ . In this paper, by using the generalized Hua's matrix inequality, the boundedness and compactness of the weighted composition operators  $W_{\varphi,u}: f \rightarrow u f \circ \varphi$  on Bers-type space of the first kind Cartan-Hartogs domain are characterized.

**Keywords:** Weighted composition operator; Generalized Hua's matrix inequality; First kind Cartan-Hartogs domain; Bers-type space; Boundedness; Compactness (2010 MSC 32A37,47B33)

#### 1 Introduction

The Bergman kernel function plays an important role in several complex variables. But, for which domains can the Bergman kernel function be computed by explicit formulas? In general, it is difficult to get the domain whose Bergman kernel function can be gotten explicitly. It is well known that all irreducible bounded symmetric domains were divided into six types by Cartan in 1930s. The first four types of irreducible domains are called the classical bounded symmetric domains as well as the rest two types are called exceptional domains consisting of two domains (16 and 27 dimensional domain, respectively). In 1998, Yin and Roos introduced four kinds of do-

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mains corresponding with the classical bounded symmetric domains and called this four kinds of domains as Cartan-Hartogs domains<sup>[1-27]</sup>. Yin has obtained the explicit Bergman kernel functions for the first Cartan-Hartogs domain in Ref. [26].

In this paper, we study the firstkind Cartan-Hartogs domain denoted by  $Y_I(N, m, n; K)$ , which is expressed by the following form:

$$Y_I(N,m,n;K) = \{W \in \mathbb{C}^N, Z \in \mathfrak{R}_I(m,n): \ |W|^{2K} < \det(I - Z\bar{Z}^T)\},$$

where K>0 and  $\Re_I(m,n)$  denotes

$$\Re_I(m,n) = \{Z: Z \in \mathbb{C}^{m \times n}, I - Z \overline{Z}^T > 0\}.$$

Here  $\overline{Z}$  denotes the conjugate of the matrix Z,  $Z^T$  denotes the transpose of Z, and m, n are positive integers. We denote  $Y_I(N, m, n; K)$  by  $Y_I$  if no ambiguity can arise.

Let  $\Omega$  be a domain of  $\mathbf{C}^n$  and  $H(\Omega)$  the class of all holomorphic functions on N. Let  $\varphi$  be a holomorphic self-map of  $\Omega$  and  $u \in H(\Omega)$ . The weighted composition operator  $W_{\varphi, u}$  on some subspaces of  $H(\Omega)$  is defined by

$$W_{\varphi,u}f(z) = u(z)f(\varphi(z)), z \in \Omega.$$

If u=1, it becomes the composition operator, usually denoted by  $C_{\varphi}$ . If  $\varphi(z)=z$ , it becomes the multiplication operator, usually denoted by Mu. A standard problem is to provide function theoretic characterizations when  $\varphi$  and u induce a bounded or compact weighted composition operator. In recent years, there is a great interest in the weighted composition operators on or between spaces of the various bounded domains (see, e. g., Refs. [3, 5, 12, 15, 16, 19] for the unit disk, Refs. [11, 17, 18, 20] for the unit ball, Refs. [13, 23, 24] for the unit poly disk, Refs. [1, 2] for the bounded homogeneous domain and Refs. [8, 10, 21, 22] for the half-plane).

Let  $\alpha > 0$  and  $\mathbb{B} = \{z \in \mathbf{C}^n : |z| < 1\}$  be the open unit ball of  $\mathbf{C}^n$ . The well-known Bers-type space on  $\mathbb{B}$ , usually denoted by  $A_{\alpha}(\mathbb{B})$ , consists of all  $f \in (\mathbb{B})$  such that

$$\parallel f \parallel = \sup_{z \in \mathbb{B}} (1 - |z|^2)^{\alpha} |f(z)| < + \infty.$$

For the Bers-type spaces and some concrete operators on them, see, for instance, Refs. [6, 7, 9,

27] and the references therein.

Let  $\alpha > 0$ , K > 0. Following the definition of Bers-type space on  $\mathbb{B}$ , we say that  $f \in H(Y_I)$  is in the Bers-type space  $A_{\alpha}(Y_I)$  if

$$||f||_{A_{\alpha}(Y_I)} = \sup_{(Z,W) \in Y_I} \left[ \det(I - Z\bar{Z}^T) - |W|^{2K} \right]^{\alpha} |f(Z,W)| < +\infty.$$

It is not difficult to see that  $A_{\alpha}(Y_I)$  is a Banach space under the quantity  $\| \cdot \|_{A_{\alpha}(Y_I)}$ .

Motivated by the results of weighted composition operators on holomorphic function spaces of the classical domains, we study this kinds of operators on Bers-type space of the first Cartan-Hartogs domain and characterize the boundedness and compactness.

Throughout this paper, the constants are denoted by C, which is positive and may differ from one occurrence to the next.

### 2 Preliminaries

First, we have the following easy result.

**Lemma 2.1** Let  $\alpha > 0$ , K > 0. For each  $(Z, W) \in Y_I$  and  $f \in A_{\alpha}(Y_I)$ , it follows that

$$|f(Z,W)| \leq \frac{||f||_{A_{\alpha}(Y_I)}}{\left[\det(I - Z\bar{Z}^T) - |W|^{2K}\right]^{\alpha}}.$$

In order to characterize the compactness, we need the following result which is similar to Proposition 3. 11 in Ref. [4].

**Lemma 2.2** Let  $\alpha > 0$ , K > 0,  $\varphi$  be a holomorphic self-map of  $Y_I$  and  $u \in H(Y_I)$ . Then the bounded operator  $W_{\varphi, u}$  is compact on  $A_{\alpha}(Y_I)$  if and only if for every bounded sequence  $\{f_k\}$  in  $A_{\alpha}(Y_I)$  such that  $f_k \rightarrow 0$  uniformly on every compact subset of  $Y_I$  as  $k \rightarrow \infty$ , it follows that

$$\lim_{k \to \infty} \| \mathbf{W}_{\varphi, u} f_k \|_{A_\alpha(Y_I)} = 0 \tag{1}$$

**Proof** Assume that the bounded operator  $W_{\varphi,u}$  on  $A_{\alpha}(Y_I)$  is compact. Let  $\{f_k\}$  be abounded sequence in  $A_{\alpha}(Y_I)$  such that  $f_k \rightarrow 0$  uniformly on every compact subset of  $A_{\alpha}(Y_I)$  as  $k \rightarrow \infty$ . If  $\|W_{\varphi,u}f_k\|_{A_{\alpha}(Y_I)} \rightarrow 0$  as  $k \rightarrow \infty$ , then there exists a subsequence  $\{f_{k_i}\}$  of  $\{f_k\}$  such that

$$\inf_{j \in \mathbf{N}} \| W_{\varphi, u} f_{k_j} \|_{A_{\alpha}(Y_I)} > 0.$$

Since  $W_{\varphi, u}$  is compact on  $A_{\alpha}(Y_I)$ , there exist a function  $g \in A_{\alpha}(Y_I)$  and a subsequence of  $\{f_{k_i}\}$ 

(without loss of generality, still written by  $\{f_{k_i}\}$ ), such that

$$\lim_{i \to \infty} \| W_{\varphi,u} f_{k_j} - g \|_{A_a(Y_I)} = 0.$$

Let E be a compact subspace of  $Y_I$ . For  $(Z,W) \in E$ , from Lemma 2.1 it follows that

$$\left| \left( W_{\varphi,u} f_{k_{j}} - g \right) (Z, W) \right| \leqslant$$

$$\frac{ \| W_{\varphi,u} f_{k_{j}} - g \|_{A_{\alpha}(Y_{I})}}{ \left\lceil \det \left( I - \overline{Z} Z^{T} \right) - |W|^{2K} \right\rceil^{\alpha}}$$

$$(2)$$

From (2), we see that  $W_{\varphi,u}f_{k_j} - g \rightarrow 0$  uniformly on E as  $j \rightarrow \infty$ . From this, for arbitrary  $\varepsilon > 0$ , there exists a positive integer  $N_1$  such that

$$|u(Z,\xi)f_{k_j}(\varphi(Z,W))-g(Z,W)| < \varepsilon$$
 (3) for all  $(Z,W) \in E$ , whenever  $j > N_1$ . Since  $f_{k_j} \to 0$  uniformly on  $E$  as  $j \to \infty$ , also there exists a positive integer  $N_2$  such that  $|f_{k_j}(Z,W)| < \varepsilon$  for all  $(Z,W) \in E$ , whenever  $j > N_2$ .

Let

$$N = \max\{N_1, N_2\}$$

and

$$M = \max_{(Z,W)\in E} |u(Z,W)|.$$

From (3), we have

$$|g(Z,W)| \leq |f_{k_{j}}(\varphi(Z,W))| \max_{(Z,W)\in K} |u(Z,W)| + \epsilon = (1+M)\epsilon$$
(4

for all  $(Z, W) \in E$ , whenever j > N. From (4) and the arbitrariness of  $\varepsilon$ , we obtain g(Z, W) = 0 for all  $(Z, W) \in E$ , which leads to  $g \equiv 0$  on  $Y_I$ . This shows that

$$\lim_{i\to\infty}\|W_{\varphi,u}f_{k_j}\|_{A_\alpha(Y_I)}=0,$$

which contradicts (1).

Now, assume that  $\{f_k\}$  is a bounded sequence in  $A_{\alpha}(Y_I)$ . Then it is locally uniform bounded on  $Y_I$ , which shows that there exists a subsequence  $\{f_{k_j}\}$  of  $\{f_k\}$  such that  $f_{k_j} \rightarrow f$  uniformly on every compact subset of  $Y_I$  as  $j \rightarrow \infty$ . From this we have  $f_{k_j} - f \rightarrow 0$  uniformly on every compact subset of  $Y_I$  as  $j \rightarrow \infty$ . Consequently, we obtain

$$\begin{split} &\lim_{j\to\infty} \| \operatorname{W}_{\varphi,u}(f_{k_j} - f) \|_{A_\alpha(Y_I)} = \\ &\lim_{j\to\infty} \| \operatorname{W}_{\varphi,u} f_{k_j} - \operatorname{W}_{\varphi,u} f \|_{A_\alpha(Y_I)} = 0 \,, \end{split}$$

which shows that  $W_{\varphi,u}$  is compact on  $A_{\alpha}(Y_I)$ .

In the studies of the several complex variables, the famous mathematician Hua found and proved the following so-called Hua's matrix inequality in 1955:

If A, B are  $n \times n$  complex matrices, and  $I - A\bar{A}^T$  and  $I - B\bar{B}^T$  are both Hermitian positive definite matrices, then

$$\det(I - A \bar{A}^T) \det(I - B \bar{B}^T) \leq |\det(I - A \bar{B}^T)|^2.$$

Recently, Ref. [25] gives a generalization of Hua's inequality in  $Y_I$ .

**Lemma 2.3** Let K > 0. For any two points  $(Z_1, W_1)$ ,  $(Z_2, W_2) \in Y_I$ , it follows that

$$\left[ \det(I - Z_1 \overline{Z_1}^T) - |W_1|^{2K} \right] \left[ \det(I - Z_2 \overline{Z_2}^T) - |W_2|^{2K} \right] \leq \left| \det(I - Z_1 \overline{Z_2}^T) - (W_1 \overline{W_2}^T)^K \right|^2.$$

Applying Lemma 2. 3, we obtain some useful functions in  $A_a(Y_I)$ .

**Lemma 2.4** Let  $\alpha$ , K > 0. For each fixed point  $(A,B) \in Y_I$ , the function

$$f_{(A,B)}(Z,W) = \frac{\left[\det(I - \bar{A}A^T) - |B|^{2K}\right]^{\alpha}}{\left[\det(I - \bar{Z}A^T) - (\bar{W}B^T)^K\right]^{2\alpha}}$$

belongs to  $A_{\alpha}(Y_I)$ , and

$$\sup_{(A,B)\in Y_I} \| f_{(A,B)} \|_{A_a(Y_I)} \leq 1 \tag{5}$$

**Proof** From a direct calculation and Lemma 2.3, we have

$$\left[ \det(I - \overline{Z}Z^{T}) - |W|^{2K} \right]^{\alpha} |f_{(A,B)}(Z,W)| =$$

$$\left[ \det(I - \overline{Z}Z^{T}) - |W|^{2K} \right]^{\alpha} \cdot$$

$$\left[ \det(I - \overline{A}A^{T}) - |B|^{2K} \right]^{\alpha}$$

$$\left| \det(I - \overline{Z}A^{T}) - (\overline{W}B^{T})^{K} \right|^{2\alpha}$$

From this, we obtain  $f_{(A,B)} \in A_{\alpha}(Y_I)$  and (5) holds.

#### 3 The main results

For  $\varphi$  a holomorphic self-map of  $Y_I$ , let  $(A,B) = \varphi(Z,W)$  for  $(Z,W) \in Y_I$ .

**Theorem 3.1** Let  $\alpha > 0$ , K > 0,  $\varphi$  the holomorphic self-map of  $Y_I$  and  $u \in H(Y_I)$ . Then the operator  $W_{\varphi, u}$  is bounded on  $A_{\alpha}(Y_I)$  if and only if

$$M_I \coloneqq \sup_{(Z,W) \in Y_I} |u(Z,W)|$$

$$\frac{\lceil \det (I - \overline{Z}Z^{\scriptscriptstyle \mathsf{T}}) - |W|^{2K} \rceil^a}{\lceil \det (I - \overline{A}A^{\scriptscriptstyle \mathsf{T}}) - |B|^{2K} \rceil^a} \! < \! + \! \infty.$$

**Proof** Assume that  $W_{\varphi,u}$  is bounded on  $A_{\alpha}(Y_I)$ . Then for each  $f \in A_{\alpha}(Y_I)$ , there exists a positive constant Csuch that

$$\|W_{\varphi,u}f\|_{A_{q}(Y_{I})} \leq C \|f\|_{A_{q}(Y_{I})}$$
 (6)

(8)

For the fixed point  $(Z_1, W_1) \in Y_I$ , choose the function

$$f_{(A,B)}(Z,W) = \frac{\left[\det(I - \bar{A}A^T) - |B|^{2K}\right]^{\alpha}}{\left[\det(I - \bar{Z}A^T) - (\bar{W}B^T)^K\right]^{2\alpha}},$$

where  $(A,B) = \varphi(Z_1,W_1)$ . From Lemma 2.4, it follows that  $f_{(A,B)} \in A_{\alpha}(Y_I)$ . and  $\|f_{(A,B)}\|_{A_{\alpha}(Y_I)} \le 1$ . By a direct computation, we have

$$|f_{(A,B)}(\varphi(Z_1, W_1))| = \frac{\left[\det(I - Z_1 \overline{Z}_1^T) - |W_1|^{2K}\right]^{\alpha}}{\left[\det(I - \overline{A}A^T) - |B|^{2K}\right]^{\alpha}}$$
(7)

Then, applying the boundedness of  $W_{\varphi,u}$  to  $f_{(A,B)}$  along with (6) and (7), we have

$$\left[ \det(I - Z_1 \overline{Z_1}^T) - |W_1|^{2K} \right]^{\alpha} |W_{\varphi,u} f_{(A,B)} (Z_1, W_1)| = \frac{\left[ \det(I - Z_1 \overline{Z_1}^T) - |W_1|^{2K} \right]^{\alpha}}{\left[ \det(I - \overline{A} \overline{A}^T) - |B|^{2K} \right]^{\alpha}} |u(Z_1, W_1)| \leq W_{\varphi,u} f_{(A,B)} \| \leq C \| f_{(A,B)} \|_{A_{\alpha}(Y_I)} \leq C,$$

which shows that

$$M_I =$$

$$\sup_{(Z,W)\in Y_I}\frac{\lceil\det(I-\bar{Z}Z^T)-|W|^{2K}\rceil^a}{\lceil\det(I-\bar{A}A^T)-|B|^{2K}\rceil^a}|u(Z,W)|<$$

$$+\infty$$

Conversely, by Lemma 2.1, for all  $f \in A_{\alpha}(Y_I)$  we have

$$\begin{split} \sup_{(Z,W)\in Y_I} \left[ \det(I - \bar{Z}Z^T) - |W|^{2K} \right]^a \left| W_{\varphi,u} f(Z,W) \right| &= \\ \sup_{(Z,W)\in Y_I} \left[ \det(I - \bar{Z}Z^T) - |W|^{2K} \right]^a \left| u(Z,W) f(\varphi(Z,W)) \right| &\leq \\ \sup_{(Z,W)\in Y_I} \left| u(Z,W) \right| \frac{\left[ \det(I - \bar{Z}Z^T) - |W|^{2K} \right]^a}{\left[ \det(I - \bar{A}A^T) - |B|^{2K} \right]^a} \parallel f \parallel_{A_a(Y_I)} &= M_I \parallel f \parallel_{A_a(Y_I)} \end{split}$$

Hence, from (8) we see that the operator  $W_{\varphi,u}$  is bounded on  $A_{\alpha}(Y_I)$ . The proof is end.

Next, we characterize the compactness of the operator  $W_{\varphi,u}$  on  $A_{\alpha}(Y_I)$ .

**Theorem 3.2** Let  $\alpha>0$ , K>0,  $\varphi$  the holomorphic self-map of  $Y_I$  and  $u\in H(Y_I)$ . Then the operator  $W_{\varphi,u}$  is compact on  $A_{\alpha}(Y_I)$  if and only if

$$\lim_{(A,B)\to\partial Y_I} |u(Z,W)| \frac{\left[\det(I-\bar{Z}Z^T)-|W|^{2K}\right]^{\alpha}}{\left[\det(I-\bar{A}A^T)-|B|^{2K}\right]^{\alpha}}$$

$$=0 \tag{9}$$

**Proof** Assume that the operator  $W_{\varphi,u}$  is compact on  $A_{\alpha}(Y_I)$ . Then it is clear that the operator  $W_{\varphi,u}$  is bounded on  $A_{\alpha}(Y_I)$ . Consider a sequence  $\{(A_i,B_i)\}=\{\varphi(Z_i,W_i)\}$  in  $Y_I$  such that  $(A_i,B_i)\rightarrow\partial Y_I$  as  $i\rightarrow\infty$ . If such a sequence does not exist, then (9) obviously holds. Using this sequence, we define the function sequence

$$f_i(Z, W) = f_{(A_i, B_i)}(Z, W),$$

where  $f(A_i, B_i)$  is the function  $f_{(A,B)}$  replaced (A,B) by  $(A_i,B_i)$  in the proof of Theorem 3.1.

By Lemma 2.4, we see that the sequence 
$$\{f_i\}$$
 is uniformly bounded in  $A_a(Y_I)$ , and  $f_i \to 0$  uniformly on any compact subset of  $Y_I$  as  $i \to \infty$ . So by Lemma 2. 2 one has  $\lim_{i \to \infty} \|W_{\varphi,u}f_i\|_{A_a(Y_I)} = 0$ . From this and a direct calculation, we have 
$$\lim_{i \to \infty} |u(Z_i, W_i)| \frac{\lceil \det(I - Z_i \overline{Z_i}^T) - |W_i|^{2K} \rceil^a}{\lceil \det(I - A_i \overline{A_i}^T) - |B_i|^{2K} \rceil^a} = 0.$$

Conversely, in order to prove that the operator  $W_{\varphi,u}$  is compact on  $A_{\alpha}(Y_I)$ , by Lemma 2. 2 we only need to prove that, if  $\{f_i\}$  is a sequence in  $A_{\alpha}(Y_I)$  such that  $\sup_{i\in \mathbf{N}} \|f_i\|_{A_{\alpha}(Y_I)} \leqslant M$  and  $f_i \to 0$  uniformly on any compact subset of  $Y_I$  as  $i \to \infty$ , then  $\lim_{i\to\infty} \|W_{\varphi,u}f_i\|_{A_{\alpha}(Y_I)} = 0$ . We first observe that (9) implies that for every  $\varepsilon > 0$ , there exists an  $\sigma > 0$ , such that for any

$$\begin{split} &(Z,W) \in E = \\ &\{(Z,W) \in Y_I : \operatorname{dist}(\varphi(Z,W), \partial Y_I) <_{\sigma}\}, \\ &|u(Z,W)| \frac{\left[\det(I - \bar{Z}Z^T) - |W|^{2K}\right]^{\alpha}}{\left[\det(I - \bar{A}A^T) - |B|^{2K}\right]^{\alpha}} <_{\varepsilon} (10) \end{split}$$

For such  $\varepsilon$  and  $\sigma$ , by using (10) and Lemma 2.1,

we have

$$\begin{split} \| \, W_{\varphi,u} f_i \, \|_{A_\alpha(Y_I)} &= \sup_{(Z,W) \in Y_I} \left[ \det \left( I - \bar{Z} Z^T \right) - |W|^{2K} \right]^\alpha \left| \, W_{\varphi,u} f_i(Z,W) \, \right| = \\ &= \sup_{(Z,W) \in Y_I} \left[ \det \left( I - \bar{Z} Z^T \right) - |W|^{2K} \right]^\alpha \left| \, u(Z,W) f_i(\varphi(Z,W)) \, \right| \leqslant \\ &\left( \sup_{(Z,W) \in E} + \sup_{(Z,W) \in Y_I \setminus E} \right) \left[ \det \left( I - \bar{Z} Z^T \right) - |W|^2 \right]^\alpha \left| \, u(Z,W) f_i(\varphi(Z,W)) \, \right| \leqslant \\ &M_{\mathfrak{E}} + \sup_{(Z,W) \in Y_I \setminus E} \left[ \det \left( I - \bar{Z} Z^T \right) - |W|^{2K} \right]^\alpha \left| \, u(Z,W) f_i(\varphi(Z,W)) \, \right| \leqslant \\ &M_{\mathfrak{E}} + \sup_{(Z,W) \in Y_I \setminus E} \left[ \det \left( I - \bar{Z} Z^T \right) - |W|^{2K} \right]^\alpha \left| \, u(Z,W) \, |_{(Z,W) \in Y_I \setminus E} \, \right| f_i(\varphi(Z,W)) \, \big| \, . \end{split}$$

Since  $\{(Z,W) \in Y_I \setminus E\}$  is a compact subset of  $Y_I$ ,  $f_i \rightarrow 0$  uniformly on this set as  $i \rightarrow \infty$ . From this and (11) we get

$$\lim_{i \to \infty} \| W_{\varphi,u} f_i \|_{A_{\alpha}(Y_I)} = 0,$$

which shows that the operator  $W_{\varphi,u}$  is compact on  $A_{\alpha}(Y_I)$ .

## 4 Conclusions

We study the weighted composition operators on Bers-type space of the first kind Cartan-Hartogs domain and characterize the boundedness and compactness by using the generalized Hua's matrix inequality.

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