

一种计算 Tchebyscheff 映射 高阶关联函数的数论方法

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摘要: 关联函数是混沌映射的统计理论的核心. 本文主要研究 Tchebyscheff 映射的高阶关联函数的计算问题. 对此问题, 已有 Beck 于 1991 年提出的一种图论方法. 然而, 当映射和关联函数的阶都比较大时该方法非常低效. 本文基于 Tchebyscheff 映射关联函数的定义提出了一种数论方法. 该方法将关联函数的计算问题转化为一类具有严格单调递增指数的指数型丢番图方程的求解问题, 进而逐级地求得方程的解. 然后, 本文研究了当映射的阶不小于关联函数的阶时非零关联函数的计算问题. 计算结果显示, 此时关联函数的值不依赖于映射的阶, 且非零关联函数的个数与第二类斯特林数密切相关. 作为应用本文最后计算了满足条件的所有 12 阶非零关联函数的值.

关键词: 关联函数; Tchebyscheff 映射; 指数型丢番图方程

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A number theoretical method for calculating high order correlation functions of the Tchebyscheff maps

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Abstract: Correlation functions play a key role in the statistical description of chaotic maps. The main concern of this paper is the calculation of correlation functions of the Tchebyscheff maps, which is traditionally handled by using a graph theoretical method introduced by Beck in 1991. However, this method has poor efficiency when the orders of map and correlation function are large. To overcome this problem, we introduce a number theoretical method based on the definition of correlation functions of the Tchebyscheff maps. In this method, the calculation is transformed into solving a class of Diophantine equations with strictly increasing exponentials, which can be solved in a hierarchical way. Then we obtain all non-vanishing correlation functions with order not more than the order of map and show that the value of correlation functions is independent of the order of map as well as the number of non-vanishing correlation functions is closely related to the Stirling numbers of the second kind. As an application, we calculate all non-vanishing 12-order correlation functions of the maps with order no less than 12.

Keywords: Correlation function; Tchebyscheff map; Exponential Diophantine equation
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1 Introduction

Correlation functions play a key role in statistical theory of deterministic systems^[1]. Meanwhile, correlation functions are also the center of statistical description of chaotic maps, such as the Bernoulli shift map, tent map, logistic map and Tchebyscheff map, to name but a few. For instance, the existence of non-vanishing odd correlation functions defines the statistical symmetry^[2,3] of a chaotic map.

For a dynamical system subjected to additive or multiplicative chaotic noise generated by a chaotic map, all non-vanishing correlation functions of the system variables are needed to fully understand the dynamical behaviors of the system due to the non-Gaussian chaotic noise. Furthermore, these correlation functions can be calculated by taking correlation functions of the chaotic map as a prior. However, to our best knowledge, there is still no universal method for calculating non-vanishing correlation functions of a general chaotic map, besides the Tchebyscheff maps.

A Tchebyscheff map $T_{N \geq 2}$ is defined by the Tchebyscheff polynomial

$$T_N(y) = \cos(\text{Narccos } y) \quad (1)$$

where the order N is a natural number. For example, $T_2(y) = 2y^2 - 1$ is the famous Ulam map, say, the inverse of logistic map with parameter 2. It is well known that every $T_{N \geq 2}$ is semi-conjugated to a Bernoulli shift^[4], hence is ergodic, mixing and has the following natural invariant density

$$\rho(y) = \frac{1}{\pi \sqrt{1-y^2}}, y \in [-1, 1] \quad (2)$$

independent of N . Statistically, a r -order correlation function of $T_{N \geq 2}$ is defined on a sequence of iterative values $\{y_{n_i}\}_{i=1}^r$ as^[5,6]

$$\langle y_{n_1} y_{n_2} \cdots y_{n_r} \rangle = \int_{-1}^1 y_{n_1} y_{n_2} \cdots y_{n_r} \rho(y_0) dy_0 \quad (3)$$

where $y_{n_i} = T_N^{n_i}(y_0)$ is the n_i -th iteration of the arbitrarily chosen initial value y_0 . Due to the symmetry of (3), one can always set $1 \leq n_1 \leq \cdots \leq n_i \leq \cdots \leq n_r$. In such way, $\{y_{n_i}\}_{i=1}^r$ becomes to an ordered sequence. Furthermore, by setting

$y_0 = \cos \pi u, u \in [0, 1]$, (3) can be rewritten into

$$\langle y_{n_1} y_{n_2} \cdots y_{n_r} \rangle = 2^{-r} \sum_{\lambda} \delta(\lambda_1 N^{n_1} + \lambda_2 N^{n_2} + \cdots + \lambda_r N^{n_r}, 0), \lambda_i = \pm 1 \quad (4)$$

where $\sum_{\lambda} \cdot$ denotes the summation runs over all possible tuples of $(\lambda_1, \lambda_2, \cdots, \lambda_r)$ and $\delta(i, j)$ is the Kronecker delta function defined as

$$\delta(i, j) = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases}$$

In order to calculate the correlation functions of $T_{N \geq 2}$ by using (4), Beck^[5] in 1991 introduced a graph theoretical method and calculated all non-vanishing correlation functions of T_2 with order $r \leq 4$. Meanwhile, he also proved that $\langle y_n' \rangle = 0$ for all odd r . However, this method is efficient just for small N and r . When both N and r are large, this method has poor efficiency, even be impractical. Here we mention that Hilgers and Beck^[6] calculated the correlation functions of $T_{N \geq 2}$ also by this method, therein complicated discussion was involved.

In this paper, we introduce a number theoretical method for calculating high order correlation functions of $T_{N \geq 2}$ for large N and r . This method is a direct extension of that introduced in Ref. [7], therein the method is mainly utilized to calculate non-vanishing correlation functions of T_2 . In this method, starting from (4), we transform the calculation into solving a class of Diophantine equations with monotonic exponentials. Then the monotonicity is sharpened to strictly monotonicity by merging the common terms in the equation. Finally, non-vanishing correlation functions are calculated by solving these exponential Diophantine equations in a hierarchical way. As an application of this method, we calculate all non-vanishing correlation functions of $T_{N \geq 2}$ with order $r \leq N$. At the end of this paper, we take $r = 12 \leq N$ as an example to illuminate the efficiency of the method.

This paper is arranged as follows. In Section 2, we introduce the exponential Diophantine equations. In Section 3, we calculate all non-vanishing correlation functions with order $r \leq N$. Then

we calculate all non-vanishing 12-order correlation functions of $T_{N \geq 12}$ in Section 4. Finally, we conclude our findings in Section 5.

2 The exponential Diophantine equations

Given an ordered tuple (n_1, n_2, \dots, n_r) , the value of $\langle y_{n_1} y_{n_2} \dots y_{n_r} \rangle$ is calculated by counting the number of all different tuples of $(\lambda_1, \lambda_2, \dots, \lambda_r)$ satisfying

$$\sum_{i=1}^r \lambda_i N^{n_i} = 0, \lambda_i = \pm 1 \quad (5)$$

in (4) and then multiplying 2^{-r} . In other words, (n_1, n_2, \dots, n_r) can be seen as a solution of the equation

$$\sum_{i=1}^r \lambda_i N^{x_i} = 0, \lambda_i = \pm 1, 1 \leq x_1 \leq x_2 \leq \dots \leq x_r \quad (6)$$

for the tuple $(\lambda_1, \lambda_2, \dots, \lambda_r)$. Here we point that Eq. (6) belongs to a class of Diophantine equations having increasing exponentials. Then, based on the monotonicity of the exponentials in Eq. (6), we have the following two results.

Proposition 2.1 Eq. (6) has no solution for odd N and r .

Proof Suppose that Eq. (6) has a solution (n_1, n_2, \dots, n_r) for the tuple $(\lambda_1, \lambda_2, \dots, \lambda_r)$ when N and r are odd numbers. Then one has $\sum_{i=1}^r \lambda_i N^{n_i} =$

0. It follows that $\sum_{i=1}^r \lambda_i N^{n_i} \equiv \sum_{i=1}^r \lambda_i \equiv 0 \pmod{2}$ since N is odd. But this is impossible since r is also odd. The proof is end.

Recall that a chaotic map is called to be statistical symmetric if all its odd correlation functions vanish. Thus Proposition 2.1 simply de-claim that all odd $T_{N \geq 2}$ are statistical symmetric.

Proposition 2.2 Eq. (6) has no solution when $1 \leq x_1 < \dots < x_i < \dots < x_r$, which means that $\langle y_{n_1} y_{n_2} \dots y_{n_r} \rangle = 0$ when $1 \leq n_1 < n_2 < \dots < n_r$.

The proof of Proposition 2.2 is trivial, we omit it here. Proposition 2.2 simply states that some common terms must exist in the sequence $y_{n_1}, y_{n_2}, \dots, y_{n_r}$ of iterative values resulting non-

vanishing correlation function $\langle y_{n_1} y_{n_2} \dots y_{n_r} \rangle$ of $T_{N \geq 2}$. Therefore, after merging these common terms in the set $\{y_{n_1}, y_{n_2}, \dots, y_{n_r}\}$, $\langle y_{n_1} y_{n_2} \dots y_{n_r} \rangle$ can be rewritten into a new form $\langle y_{n_1}^{k_1} y_{n_2}^{k_2} \dots y_{n_s}^{k_s} \rangle$, where $1 \leq n_1 < n_2 < \dots < n_s$, and at least one $k_i \geq 2, i=1, 2, \dots, s$.

Correspondingly, Eq. (6) can be rewritten. Suppose that the common terms in Eq. (6) are

$$\begin{aligned} x_1 &= \dots = x_{k_1}, \\ x_{k_1+1} &= \dots = x_{k_1+k_2}, \\ &\dots, \\ x_{k_1+k_2+\dots+k_{s-1}+1} &= \dots = x_{k_1+k_2+\dots+k_s} = x_r. \end{aligned}$$

Let

$$\begin{aligned} M_1 &= k_1, M_i = \sum_{j=1}^i k_j, i = 2, 3, \dots, s-1, \\ M_s &= \sum_{i=1}^s k_i = r \end{aligned} \quad (7)$$

$$z_1 = x_1, z_i = x_{M_{i-1}+1}, i = 2, 3, \dots, s \quad (8)$$

and

$$\sigma_1 = \sum_{j=1}^{k_1} \lambda_j, \sigma_i = \sum_{j=1}^{k_i} \lambda_{M_{i-1}+j}, i = 2, 3, \dots, s \quad (9)$$

Eq. (6) can be rewritten into

$$\sum_{i=1}^s \sigma_i N^{z_i} = 0, 1 \leq z_1 < z_2 < \dots < z_s \quad (10)$$

Eq. (10) with (7)~(9) is also a Diophantine equation, but with strictly increasing exponentials. Meanwhile, we also have $0 \leq |\sigma_i| \leq k_i, i=1, 2, \dots, s$ from (9).

Lemma 2.3^[7] Let $\sigma_i, i=1, 2, \dots, s$ be defined as (9). We have

(i) When $2 \nmid k_i$, σ_i runs over $\{\pm k_i, \pm(k_i-2), \dots, \pm(k_i-2m_i), \dots, \pm 1\}$ and thus $2 \nmid \sigma_i$;

(ii) When $2 \mid k_i$, σ_i runs over $\{\pm k_i, \pm(k_i-2), \dots, \pm(k_i-2m_i), \dots, \pm 2, 0\}$ and thus $2 \mid \sigma_i$,

where $m_i=0, 1, \dots, k_i$ represents the number of -1 in all of the summands of (9).

Definition 2.4 $z_i=n_i, i=1, 2, \dots, s$ is called the trivial solution of Eq. (10) for $\sigma_i=0, i=1, 2, \dots, s$.

Theorem 2.5 If $N \nmid \sigma_1$ then Eq. (10) has no solution.

Proof We show this by making a contradiction. Assume that $N \nmid \sigma_1$ and (n_1, n_2, \dots, n_s) is a

solution of Eq. (10) for $(\sigma_1, \sigma_2, \dots, \sigma_s)$. Then $\sum_{i=1}^s \sigma_i N^{n_i} = 0, 1 \leq n_1 < \dots < n_s$. Dividing this equation by N^{n_1} yields $\sigma_1 = -\sum_{i=2}^s \sigma_i N^{n_i - n_1}$, which indicates that $N|\sigma_1$, a contradiction. The proof is end.

The following result is just a corollary of Lemma 2.3 and Theorem 2.5.

Corollary 2.6 For any odd number $r \leq N$, Eq. (10) has no solution. That is to say, every odd correlation function of $T_{N \geq 2}$ with order $r \leq N$ vanishes.

3 Non-vanishing correlation functions when $r \leq N$

Since all odd correlation functions vanish when $r \leq N$, we can set $2|r$ throughout this section. Firstly, we give the following two results without proof.

Proposition 3.1 Let $r > 0$ be an even number. Then there are $C_r^{r/2}$ different tuples of $(\lambda_1, \lambda_2, \dots, \lambda_r)$ satisfying that $\sum_{i=1}^r \lambda_i = 0, \lambda_i = \pm 1$,

where $C_n^k = \frac{n!}{k!(n-k)!}$ is a combinative number.

Proposition 3.2^[8] Let $r > 0$ be an even number and $r = \sum_{i=1}^s k_i, 2|k_i > 0$. Then (k_1, k_2, \dots, k_s) is called an even s -partition of r . There are $S(r/2, s), s = 0, 1, \dots, r/2$ different even s -partitions, where $S(n, k)$ denotes a Stirling number of the second kind.

Now we get all non-vanishing correlation functions of $T_{N \geq 2}$ with order $r \leq N$ by solving the exponential Diophantine Eq. (10). Since $r \leq N$,

$r = \sum_{i=1}^s k_i$, the discussion is divided into two cases.

Case 1. $k_1 = r = N$. In this case, $2|k_1 = N$. Eq. (10) shrinks to $\sigma_1 N^{x_1} = 0$, where $\sigma_1 = \sum_{i=1}^r \lambda_i, \lambda_i = \pm 1$. Since $0 \leq |\sigma_1| \leq N$ and Eq. (10) has solution only when $N|\sigma_1$, we declaim that Eq. (10) has solution only when $\sigma_1 = 0, \pm N$. Obviously,

Eq. (10) has only the trivial solution $x_1 = n$ when $\sigma_1 = 0$. On the other hand, when $\sigma_1 = \pm N$, Eq. (10) can be further rewritten into $\lambda_1 N^{x_1+1} = 0, \lambda_1 = \pm 1$, which has no solution. Thus we conclude that Eq. (10) has only the trivial solution $z_1 = n_1$ for $\sigma_1 = 0$. Furthermore, by Proposition 3.1, we get $C_r^{r/2}$ different tuples of $(\lambda_1, \lambda_2, \dots, \lambda_r)$ satisfying that $\sum_{i=1}^r \lambda_i = 0, \lambda_i = \pm 1$. It follows that

$$\langle y_n^r \rangle = 2^{-r} C_r^{r/2} \quad (11)$$

Case 2. $1 \leq k_i < N, i = 1, 2, \dots, s$. In this case, we have the following result.

Theorem 3.3 Let $r \leq N$ be an even number and $r = \sum_{i=1}^s k_i, 1 \leq k_i < N$. Then Eq. (10) has only the trivial solution $z_i = n_i, i = 1, 2, \dots, s$ for $\sigma_i = 0, i = 1, 2, \dots, s$ under the condition that $2|k_i, i = 1, 2, \dots, s$, where σ_i is defined by (9).

Proof Since $0 \leq |\sigma_i| < N, N|\sigma_i$ if and only if $\sigma_i = 0, i = 1, 2, \dots, s$. By Theorem 2.5, we know that Eq. (10) has solution only when $\sigma_1 = 0$.

Thus Eq. (10) shrinks to $\sum_{i=2}^s \sigma_i N^{z_i} = 0, z_2 < \dots < z_s$. Then, by repeatedly using Theorem 2.5, we get that Eq. (10) has solution only when $\sigma_i = 0, i = 1, 2, \dots, s$, which indicates that $2|k_i, i = 1, 2, \dots, s$. Finally, we conclude that Eq. (10) has only the trivial solution $z_i = n_i, i = 1, 2, \dots, s$ for $\sigma_i = 0, i = 1, 2, \dots, s$ under the condition that $2|k_i, i = 1, 2, \dots, s$. The proof is end.

Corollary 3.4 Let $r \leq N$ be an even number and $r = \sum_{i=1}^s k_i, 2|k_i, 1 \leq k_i < N, i = 1, 2, \dots, s$. Then

$$\langle y_{n_1}^{k_1} y_{n_2}^{k_2} \dots y_{n_s}^{k_s} \rangle = 2^{-r} \prod_{i=1}^s C_{k_i}^{k_i/2} \quad (12)$$

4 Applications

In this section, as an application of the number theoretical method, we calculate all non-vanishing 12-order correlation functions of $T_{N \geq 12}$. It follows that $s = 1, 2, \dots, 6$ in Proposition 3.2. The discussion can be divided into six cases.

Case 1. $s = 1$. In this case, r is actually not been partitioned and (4) turns into $\langle y_n^r \rangle$. By

(11), we have $\langle y_n^{12} \rangle = 2^{-12} \times \frac{12!}{(6!)^2}$.

Case 2. $s=2$. In this case,

$$r=12=k_1+k_2, 2|k_i, i=1, 2.$$

Then (4) turns into $\langle y_{n_1}^{k_1} y_{n_2}^{k_2} \rangle, 1 \leq n_1 < n_2$. Since $12=2+10=4+8=6+6=8+4=10+2$, we have the following results:

When $12=2+10=10+2$, one has

$$\langle y_{n_1}^2 y_{n_2}^{10} \rangle = \langle y_{n_1}^{10} y_{n_2}^2 \rangle = 2^{-12} \times 2 \times \frac{10!}{(5!)^2};$$

When $12=4+8=8+4$, one has

$$\langle y_{n_1}^4 y_{n_2}^8 \rangle = \langle y_{n_1}^8 y_{n_2}^4 \rangle = 2^{-12} \times \frac{4!}{(2!)^2} \times \frac{8!}{(4!)^2};$$

When $12=6+6$, one has

$$\langle y_{n_1}^6 y_{n_2}^6 \rangle = 2^{-12} \times \left[\frac{6!}{(3!)^2} \right]^2.$$

Case 3. $s=3$. In this case,

$$r=12=k_1+k_2+k_3, 2|k_i, i=1, 2, 3.$$

Then (4) turns into $\langle y_{n_1}^{k_1} y_{n_2}^{k_2} y_{n_3}^{k_3} \rangle, 1 \leq n_1 < n_2 < n_3$.

Since

$$12=2+2+8=2+8+2=2+4+6=2+6+4=$$

$$4+2+6=4+6+2=4+4+4=$$

$$6+2+4=6+4+2=8+2+2,$$

we have the following results:

$$\text{When } 12=2+2+8=2+8+2=8+2+2,$$

one has

$$\langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^8 \rangle = \langle y_{n_1}^2 y_{n_2}^8 y_{n_3}^2 \rangle = \langle y_{n_1}^8 y_{n_2}^2 y_{n_3}^2 \rangle = 2^{-12} \times 2^2 \times \frac{8!}{(4!)^2};$$

$$\text{When } 12=2+4+6=2+6+4=4+2+6=$$

$4+6+2=6+2+4=6+4+2$, one has

$$\langle y_{n_1}^2 y_{n_2}^4 y_{n_3}^6 \rangle = \langle y_{n_1}^2 y_{n_2}^6 y_{n_3}^4 \rangle = \langle y_{n_1}^4 y_{n_2}^2 y_{n_3}^6 \rangle =$$

$$\langle y_{n_1}^4 y_{n_2}^6 y_{n_3}^2 \rangle = \langle y_{n_1}^6 y_{n_2}^2 y_{n_3}^4 \rangle =$$

$$\langle y_{n_1}^6 y_{n_2}^4 y_{n_3}^2 \rangle = 2^{-12} \times 2 \times \frac{4!}{(2!)^2} \times \frac{6!}{(3!)^2};$$

When $12=4+4+4$, one has

$$\langle y_{n_1}^4 y_{n_2}^4 y_{n_3}^4 \rangle = 2^{-12} \times \left[\frac{4!}{(2!)^2} \right]^3.$$

Case 4. $s=4$. In this case, we have

$$12 = \sum_{i=1}^4 k_i, 2 | k_i, i = 1, 2, \dots, 4.$$

Then (4) turns into $\langle y_{n_1}^{k_1} y_{n_2}^{k_2} y_{n_3}^{k_3} y_{n_4}^{k_4} \rangle, 1 \leq n_1 < n_2 < n_3 < n_4$. Since

$$12=2+2+2+6=2+2+4+4=2+2+6+2=$$

$$2+4+2+4=2+4+4+2=$$

$$2+6+2+2=4+2+2+4=4+2+4+2=$$

$$4+4+2+2=6+2+2+2,$$

we have the following results:

When $12=2+2+2+6=2+2+6+2=2+6+2+2=6+2+2+2$, one has

$$\langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^2 y_{n_4}^6 \rangle = \langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^6 y_{n_4}^2 \rangle =$$

$$\langle y_{n_1}^2 y_{n_2}^6 y_{n_3}^2 y_{n_4}^2 \rangle = \langle y_{n_1}^6 y_{n_2}^2 y_{n_3}^2 y_{n_4}^2 \rangle =$$

$$2^{-12} \times 2^3 \times \frac{6!}{(3!)^2};$$

When

$$12=2+2+4+4=2+4+2+4=2+4+4+2=$$

$$4+2+2+4=4+2+4+2=4+4+2+2,$$

one has

$$\langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^4 y_{n_4}^4 \rangle = \langle y_{n_1}^2 y_{n_2}^4 y_{n_3}^2 y_{n_4}^4 \rangle =$$

$$\langle y_{n_1}^2 y_{n_2}^4 y_{n_3}^4 y_{n_4}^2 \rangle = \langle y_{n_1}^4 y_{n_2}^2 y_{n_3}^2 y_{n_4}^4 \rangle =$$

$$\langle y_{n_1}^4 y_{n_2}^2 y_{n_3}^4 y_{n_4}^2 \rangle = \langle y_{n_1}^4 y_{n_2}^4 y_{n_3}^2 y_{n_4}^2 \rangle =$$

$$2^{-12} \times 2^2 \times \left[\frac{4!}{(2!)^2} \right]^2.$$

Case 5. $s=5$. In this case, we have

$$12 = \sum_{i=1}^5 k_i, 2 | k_i, i = 1, 2, \dots, 5.$$

Then (4) turns into $\langle y_{n_1}^{k_1} y_{n_2}^{k_2} \cdots y_{n_5}^{k_5} \rangle, 1 \leq n_1 < n_2 < \cdots < n_5$. Since

$$12=2+2+2+2+4=2+2+2+4+2=$$

$$2+2+4+2+2=2+4+2+2+2=$$

$$4+2+2+2+2,$$

we have

$$\langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^2 y_{n_4}^2 y_{n_5}^4 \rangle = \langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^2 y_{n_4}^4 y_{n_5}^2 \rangle =$$

$$\langle y_{n_1}^2 y_{n_2}^2 y_{n_3}^4 y_{n_4}^2 y_{n_5}^2 \rangle = \langle y_{n_1}^2 y_{n_2}^4 y_{n_3}^2 y_{n_4}^2 y_{n_5}^2 \rangle =$$

$$\langle y_{n_1}^4 y_{n_2}^2 y_{n_3}^2 y_{n_4}^2 y_{n_5}^2 \rangle = 2^{-12} \times 2^4 \times \frac{4!}{(2!)^2}.$$

Case 6. $s=6$. In this case, we have

$$12 = \sum_{i=1}^6 k_i, k_i = 2, i = 1, 2, \dots, 6.$$

Then (4) turns into $\langle y_{n_1}^2 y_{n_2}^2 \cdots y_{n_6}^2 \rangle, 1 \leq n_1 < n_2 < \cdots < n_6$. Thus we have

$$\langle y_{n_1}^2 y_{n_2}^2 \cdots y_{n_6}^2 \rangle = 2^{-12} \times 2^6 = 2^{-6}.$$

5 Conclusions

We have introduced a number theoretical method for calculating high order correlation functions of $T_{\mathbb{N} \geq 2}$. We transform the calculation into solving a class of exponential Diophantine equations. To further reducing the burden of solving these equations, we deduce the corresponding Diophantine equations with strictly monotonic

exponentials.

As an application of this method, we calculate all non-vanishing correlation functions of $T_{N\geq 2}$ with order $r\leq N$. It is shown that, while the odd correlation functions always vanish, the even correlation functions are always non-vanishing and have values independent of N . We further calculate all non-vanishing 12-order correlation functions of $T_{N\geq 12}$ to illuminate the efficiency of this method.

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