doi：103969/j. issn. 0490-6756. 2016. 11. 017

运动粘滞大气中的耦合方程组解析解的新方法

纪娟娟[1,2]，郭业才[3]，张兰芳[2]

(1. 南京信息工程大学大气科学学院，南京 210044；2. 安庆师范学院物理与电气工程学院，安庆 246133；
3. 南京信息工程大学电子与信息工程学院,南京 210044)

摘　要：由于声波在大气中传播特性处理的数学复杂性,求耦合方程组解析解的工作已很少见. 本文首次将 NMA(Normal Mode Analysis)和同伦分析方法(HAM，Homotopy Analysis Method)相结合对考虑风和粘滞因素的耦合方程组进行解析解的求解. 首先由基本控制方程推导了运动粘滞大气中的耦合方程组,通过匀质无风的耦合方程组,对声波在大气中衰减特性和相速度进行了分析,之后利用 NMA 对其进行了解析解求解,并将其作为初始近似,利用同伦分析方法对有风、粘滞分层大气中的耦合方程组进行了三阶近似解析解的求解,最后进行了数值模拟. 结果表明,由于多种大气要素的影响,随着传播距离的增加,声压峰值越小,且频率越大衰减越快,因此风和粘滞特性是影响近地面声波传播特性的重要因素.

关键词：Normal Mode Analysis；同伦分析方法；粘滞衰减；相速度

中图分类号：02. 30. MV　　**文献标识码**：A　　**文章编号**：0490-6756(2016)06-1283-07

A new method for analytical solution of the coupled equations set through a windy，viscous atmosphere

JI Juan-Juan[1,2]，*GUO Ye-Cai*[3]，*ZHANG Lan-Fang*[2]

(1. College of Atmospheric Science，Nanjing University of Information Science & Technology，Nanjing 210044，China；
2. School of Physics and Electronic Engineering，Anqing Normal University，Anqing 246133，China；
3. College of Electronic and Information Engineering，Nanjing University of Information Science & Technology，Nanjing 210044，China)

Abstract：Due to mathematical complexity to process acoustic wave propagation in the atmospheric，the work of the analytic solutions for coupled equations set was very rare. Normal mode analysis and the homotopy analysis method were firstly combined to solve analytic solutions of the coupled equations set in a viscous，windy atmosphere. Firstly，the coupled equations set were deduced from the government equations，the attenuation characteristics and phase velocity were analyzed by the coupled equations set in the uniform and no wind atmosphere，and the analytical solutions of the equations set were solved by normal mode analysis,meanwhile the analytical solutions were used to initial approximations，the 3rd-order approximation analytical solutions of the coupled equations set in a windy and viscous atmosphere were obtained by homotopy analysis method. The numerical simulation indicates that due to the influence of the variety of atmospheric elements，the sound pressure peak value is smaller and bigger frequen-

收稿日期：2015-08-24

基金项目：全国优秀博士论文作者专项资金(200753)；江苏省高校自然科学基金(13KJA510001)；江苏省高校科研成果产业化推进项目(JHB 2012-9)；江苏省高校"信息与通信工程" 优势学科建设工程资助项目、江苏省普通高校研究生科研创新项目(KYLX15_0873)

作者简介：纪娟娟(1983－)，女，安徽亳州人，讲师，博士生，从事大气声学研究. E-mail:jjj0721@126. com

cies have faster attenuation with the increase of propagation distance, so the wind and viscous properties are the important factors that affect near surface acoustic wave propagation characteristics.

Keywords: Normal Mode Analysis; Homotopy analysis method; Viscous attenuation; Phase speed

1 引 言

由于声波传播特性受背景风场和温度梯度的控制,因此可以通过分析声波传播特性研究大气特性[1,2]. 由于声波在大气中传播的复杂性,数值模拟方法被广泛采用,但其不能给出解析解的表达式,且其精度有限,特别是远距离传播时,数值方法将需较大的计算量和存储量;而解析解能够给出各扰动量传播过程中的演变和基本物理图像. 此外,声波在复杂大气环境中传播时必然会有能量的耗散,在次声范围内,由于分子振动引起的吸收衰减经常小于 $10^{-5} dB\,\mathrm{km}^{-1}$,因此粘滞吸收是主导的损耗机制. 目前,已发展了多种求解非线性方程解析解的方法,如双曲正切函数法[3,4]、(G'/G)扩展方法[5-9],同伦扰动法[10],MSE(modified simple equation)方法[11-14],变分迭代法[15-18],同伦分析法[19-21],指数函数法[22-27],NMA(Normal Mode Analysis) [28-34]等. NMA 在不进行任何假设的情况下,能够求解出多种问题的精确解,同时,同伦分析方法是求解多种线性和非线性问题有效工具,其具有可调节的收敛控制因子,因此具有较高的精度,但其性能受初始值的制约,且直接影响近似解析解的精度,因此本文将 NMA 和同伦分析方法相结合,对有风和粘滞大气中的耦合方程组进行解析解求解.

本文首先由声控制方程推导出在考虑风和粘滞因素下的耦合方程组;其次,并利用在忽略风、重力的情况下的耦合方程组,对衰减及相速度进行了研究,之后利用 NMA 方法对其进行解析解求解;最后以 NMA 方法得到的解析解作为初值,对包含风和粘滞因素的耦合方程组进行同伦分析方法解析解求解;数值模拟结果表明,声压峰值随着传播距离的增大而减小.

2 耦合方程组

声压波动方程

$$\frac{Dp}{Dt} = -\rho c^2 \nabla \cdot V \tag{1}$$

可压缩粘滞流体声粒子速度方程为

$$\rho \frac{DV}{Dt} = \nabla p + \eta \nabla^2 V + (\zeta + \frac{\eta}{3}) \nabla (\nabla \cdot V) \tag{2}$$

将方程中的声压、密度、流体速度表示为背景值(使用下表 0 标注)和声扰动值(使用下表 s 标注)

$$p = p_0 + p_s, \rho = \rho_0 + \rho_s, V = w + v \tag{3}$$

式中, w 表示风速, v 是声速粒子.

作以下假设:

(1) 假设是大气是稳态平衡的,因此零阶项舍去;

(2) 仅保留线性项;

(3) $\nabla \rho_0 = \nabla p_0 = 0$;

(4) 风的散度为零 $\nabla \cdot w = 0$;

(5) 风是水平的且风的梯度沿着竖直方向.

则,方程(1)变为

$$\frac{\partial p_s}{\partial t} = -w \cdot \nabla p_s - \rho_0 c^2 \nabla \cdot v \tag{4}$$

方程(2)变为

$$\frac{\partial v}{\partial t} = -(v \cdot \nabla) w - (w \cdot \nabla) v + \frac{1}{\rho_0}(-\nabla p_s + \eta \nabla^2 (v + w) + (\zeta + \frac{\eta}{3}) \nabla (\nabla \cdot (v + w)) \tag{5}$$

为了研究粘滞引起的衰减,假设密度、风速均匀一致,且忽略风时,方程(4)、(5)变为

$$\rho_0 \frac{\partial v}{\partial t} = -\nabla p_s + (\zeta + \frac{4\eta}{3}) \nabla^2 v \tag{6}$$

$$\frac{\partial p_s}{\partial t} + \rho_0 c^2 \nabla \cdot v = 0 \tag{7}$$

对式(6)两边进行散度运算,则

$$\rho_0 \frac{\partial}{\partial t} (\nabla \cdot v) + \nabla^2 p_s - \nabla \cdot ((\zeta + \frac{4\eta}{3}) \nabla^2 v) = 0 \tag{8}$$

将式(7)两边对 t 求偏导,得

$$\frac{\partial^2 p_s}{\partial t^2} + \rho_0 c^2 \frac{\partial}{\partial t} (\nabla \cdot v) = 0 \tag{9}$$

则由式(8)、(9)可得

$$-\frac{1}{c^2} \frac{\partial^2 p_s}{\partial t^2} + \nabla^2 p_s - \nabla \cdot ((\zeta + \frac{4\eta}{3}) \nabla^2 v) = 0 \tag{10}$$

对式(7)两边取梯度

$$\frac{\partial}{\partial t} \nabla p_s + \rho_0 c^2 \nabla \nabla \cdot v = 0 \tag{11}$$

由于声场是无旋场即，$\nabla\times v=0$，因此

$$\nabla\nabla\cdot v=\nabla\times\nabla\times v+\nabla^2 v=\nabla^2 v \tag{12}$$

因此由式(11)可得，

$$\nabla^2 v=-\frac{1}{\rho_0 c^2}\frac{\partial}{\partial t}\nabla p_s \tag{13}$$

将式(13)代入式(10)，得

$$\frac{\partial^2 p_s}{\partial t^2}=c^2\left(1+q\frac{\partial}{\partial t}\right)\nabla^2 p_s \tag{14}$$

式中，

$$q=\left(\zeta+\frac{4\eta}{3}\right)/(\rho c^2) \tag{15}$$

式(14)表明内粘滞是主要的吸收机制.

假设式(14)具有如下形式的试探平面波解

$$p_s=e^{i(k\cdot r-\omega t)} \tag{16}$$

式中，k 是波数，ω 是角频率，r 是距离向量，t 是时间.

式(16)代入式(14)可得

$$k^2=\frac{\omega^2}{c^2(1-i\omega q)} \tag{17}$$

且将 k^2 表示为 $k^2=|k^2|e^{i\varphi}$.

同时波数 k 与相速度 c_p 和衰减 α 具有如下关系

$$k(\omega)=\omega/c_p(\omega)+i\alpha(\omega) \tag{18}$$

则

$$\omega/c_p(\omega)=\mathrm{Re}(k)=\sqrt{|k^2|}\cos(\varphi/2) \tag{19}$$

$$\alpha(\omega)=\mathrm{lm}(k)=\sqrt{|k^2|}\sin(\varphi/2) \tag{20}$$

利用三角函数半角公式，可以得到相速度的表达式为

$$c_p(\omega)=c\sqrt{\frac{2(1+q^2\omega^2)}{1+\sqrt{1+q^2\omega^2}}} \tag{21}$$

衰减的表达式为

$$\alpha(\omega)=\frac{\omega}{\sqrt{2}c}\sqrt{\frac{\sqrt{1+q^2\omega^2}-1}{(1+q^2\omega^2)}} \tag{22}$$

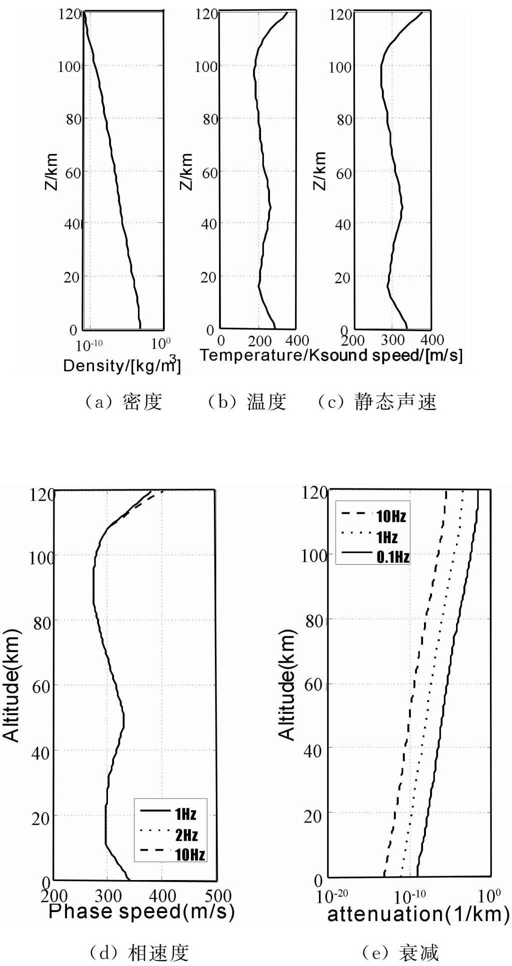
对于低于 20Hz 的次声波，$q\omega\leqslant 1$ 时，可得到近似的频散和衰减关系为

$$c_p(\omega)\approx c\left(1+\frac{3}{8}q^2\omega^2\right) \tag{23}$$

$$\alpha(\omega)\approx\frac{q\omega^2}{2c}(1-q^2\omega^2/2) \tag{24}$$

式(23)，(24)表明，对于次声，当 $q\omega\leqslant 1$，相速度和衰减与频率平方近似成正比，且随密度的减小而增加.

大气中的密度、温度和声速是不断变化的. 根据大气模式 MSISE90 计算的得到温度场和密度剖面如图 1(a)、(b)、(c)所示，以及相对应的相速度和衰减如图 1(d)、(e)所示.



(a) 密度　(b) 温度　(c) 静态声速

(d) 相速度　(e) 衰减

图 1(a) 大气密度剖面，(b) 温度，(c) 静态声速，(d) 相速度 c_p，(e) 衰减系数 $\alpha(\omega)$

Fig. 1　(a) Atmosphere density profile, (b) temperature, (c) static sound profile, (d) phase speed, (e) attenuation coefficients

由图 1(c)、(d)图可得，10Hz 的次声约在 110km 高度时，有很大的耗散，(e)图可得同一频率随着高度的增加衰减逐渐变大，在同一高度，频率越高，衰减越大.

3　NMA 方法求解

在二维坐标系下，由式(6)、式(7)和式(14)构成的系统为

$$\frac{\partial^2 p_s}{\partial t^2}=c^2\left(1+q\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 p_s}{\partial x^2}+\frac{\partial^2 p_s}{\partial z^2}\right) \tag{25}$$

$$\rho_0\frac{\partial v_x}{\partial t}+\frac{\partial p}{\partial x}-\left(\zeta+\frac{4\eta}{3}\right)\left(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial z^2}\right)=0 \tag{26}$$

$$\rho_0\frac{\partial v_z}{\partial t}+\frac{\partial p_s}{\partial z}-\left(\zeta+\frac{4\eta}{3}\right)\left(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial z^2}\right)=0 \tag{27}$$

$$\frac{\partial p_s}{\partial t}+\rho_0 c^2(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z})=0 \quad (28)$$

为了表述方便,将 p_s 下表省略. 利用 NMA (Normal Mode Analysis),变量的解可分解为

$$[v_x,v_z,p](x,z,t)=[v_x^*,v_z^*,p^*](z)e^{i(\omega t+kx)} \quad (29)$$

式中,v_x^*,v_z^*,p^* 是函数 v_x,v_z,p 的幅度,k 是 x 方向上的波数,ω 是角频率.

令 $\zeta+\frac{4\eta}{3}=b$,由式(25)~(28)分别可得到下列方程,

$$-\omega^2 p^*=c^2(1+qi\omega)(D^2p^*-k^2p^*) \quad (30)$$

$$ikp^*=bD^2v_x^*-(bk^2+i\rho\omega)v_x^* \quad (31)$$

$$Dp^*=bD^2v_z^*-(bk^2+i\rho\omega)v_z^* \quad (32)$$

$$-\rho Dv_z^*-i\rho kv_x^*=i\omega/c^2p^* \quad (33)$$

式中,$D=\frac{d}{dz}$,由式(30)可得

$$D^2p^*-m^2p^*=0 \quad (34)$$

式中,$m^2=k^2-\frac{\omega^2}{c^2(1+iq\omega)}$.

则,式(34)具有如下形式的解

$$p^*=\sum_{j=1}^{2}p_j^* \quad (35)$$

式中,$p_j^*=G_j(k,\omega)e^{r_iz}$,$r_i$ 是特征方程的根

$$r_i^2-m^2=0 \quad (36)$$

因此方程(30)的解为

$$p^*=G_1(k,\omega)e^{-mz}+G_2(k,\omega)e^{mz} \quad (37)$$

由式(29)可得,

$$v_x^*=G_1'(k,\omega)e^{-mz}+G_2'(k,\omega)e^{mz} \quad (38)$$

式中,$G_1'(k,\omega)=\frac{ik}{(bm^2-bk^2-i\rho\omega)}G_1(k,\omega)$,$G_2'(k,\omega)=\frac{ik}{bm^2-bk^2-i\rho\omega}G_2(k,\omega)$.

由式(28),可得

$$v_z^*=G_1''(k,\omega)e^{-mz}+G_2''(k,\omega)e^{mz} \quad (39)$$

式中,$G_1''(k,\omega)=-\frac{m}{bm^2-bk^2-i\rho\omega}G_1(k,\omega)$,$G_2''(k,\omega)=\frac{m}{bm^2-bk^2-i\rho\omega}G_2(k,\omega)$.

在 $z=0$ 的表面,p^*、v_z^* 满足以下边界条件,

$$v_z^*=F(x,t),\ p^*=p(x,t) \quad (40)$$

由方程(29),(37),(39)可得

$$P_0^*=G_1+G_2 \quad (41)$$

$$\frac{bm^2-bk^2-i\rho\omega}{m}F_0^*=G_2-G_1 \quad (42)$$

$$G_1=\frac{1}{2}(P_0^*-\frac{bm^2-bk^2-i\rho\omega}{m}F_0^*) \quad (43)$$

$$G_2=\frac{1}{2}(P_0^*+\frac{bm^2-bk^2-i\rho\omega}{m}F_0^*) \quad (44)$$

因此,式(13)、(14)的解为

$$p_s(x,z,t)=[G_1e^{-mz}+G_2e^{mz}]e^{i(\omega t+kx)} \quad (45)$$

$$v_z(x,z,t)=-\frac{m}{bm^2-bk^2-i\rho\omega}[G_1e^{-mz}-G_2e^{mz}]e^{i(\omega t+kx)} \quad (46)$$

$$v_x(x,z,t)=\frac{ik}{bm^2-bk^2-i\rho\omega}[G_1e^{-mz}+G_2e^{mz}]\cdot e^{i(\omega t+kx)} \quad (47)$$

又因 $z\to\infty$ 时,$p_s(x,z,t)$,$v_x(x,z,t)$,$v_z(x,z,t)$ 是有界的,因此

$$p_s(x,z,t)=G_1e^{i(\omega t+kx)-mz} \quad (48)$$

$$v_z(x,z,t)=a_1G_1e^{i(\omega t+kx)-mz} \quad (49)$$

式中,$a_1=-\frac{m}{bm^2-bk^2-i\rho\omega}$.

$$v_x(x,z,t)=a_2G_1e^{i(\omega t+kx)-mz} \quad (50)$$

式中,$a_2=\frac{ik}{bm^2-bk^2-i\rho\omega}$.

4 同伦分析方法近似解求解

令 $b_2=\zeta+\frac{1}{3}\eta$,在二维坐标系下,方程(1)、(2)展开为

$$\frac{\partial p}{\partial t}=-w_x\frac{\partial p}{\partial x}-\rho_0c^2(\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}) \quad (51)$$

$$\frac{\partial v_x}{\partial t}=-v_z\frac{\partial w_x}{\partial z}-w_x\frac{\partial v_x}{\partial x}+\frac{1}{\rho_0}(-\frac{\partial p}{\partial x}+\eta(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_x}{\partial z^2})+\eta\frac{\partial^2 w_x}{\partial z^2}+b_2(\frac{\partial^2 v_x}{\partial x^2}+\frac{\partial^2 v_z}{\partial x\partial z})) \quad (52)$$

$$\frac{\partial v_z}{\partial t}=-w_x\frac{\partial v_z}{\partial x}+\frac{1}{\rho_0}(-\frac{\partial p}{\partial z}+\eta(\frac{\partial^2 v_z}{\partial x^2}+\frac{\partial^2 v_z}{\partial z^2})+b_2(\frac{\partial^2 v_z}{\partial z^2}+\frac{\partial^2 v_x}{\partial x\partial z})) \quad (53)$$

根据同伦分析方法[35],取式(48)-(50)作为初值,即

$$p_0(x,z,t)=p_s(x,z,0)=G_1e^{ikx-mz} \quad (54)$$

$$v_{z0}(x,z,t)=v_z(x,z,0)=a_1G_1e^{ikx-mz} \quad (55)$$

$$v_{x0}(x,z,t)=v_x(x,z,0)=a_2G_1e^{ikx-mz} \quad (56)$$

设风速沿水平方向且只随高度变化,取其表达式为[36]

$$w_x=k_u[\ln(\frac{z+z_m}{z_m})+\frac{5z}{L}] \quad (57)$$

式中,$k_u=\mu_*/k$,k 是 Von Karman 常数,μ_* 摩擦速度,z_m 动量粗糙长度.

则,迭代公式为

$$p_1(x,z,t) = \hbar\int_0^t \left(\frac{\partial p_{s0}}{\partial \tau} + w_x \frac{\partial p_{s0}}{\partial x} + \rho_0 c^2 \left(\frac{\partial v_{x0}}{\partial x} + \frac{\partial v_{z0}}{\partial z}\right)\right) d\tau = \hbar t(w_x(-m)p_0 + \rho_0 c^2(ikv_{x0} + (-m)v_{z0})) = G_1 \hbar t/L e^{ikx-mz}(i5kz + Likk_u \log((z + z_m)/z_m) + ia_2 c^2 k\rho L - a_1 L c^2 m\rho)) \tag{58}$$

$$v_{x1} = \hbar\int_0^t \left(\frac{\partial v_{x0}}{\partial \tau} + v_{z0}\frac{\partial w_x}{\partial z} + w_x \frac{\partial v_{x0}}{\partial x} - \frac{1}{\rho_0}\left(-\frac{\partial p_0}{\partial x} + \eta\left(\frac{\partial^2 v_{x0}}{\partial x^2} + \frac{\partial^2 v_{x0}}{\partial z^2}\right) + \eta\frac{\partial^2 w_x}{\partial z^2} + b_2\left(\frac{\partial^2 v_{x0}}{\partial x^2} + \frac{\partial^2 v_{z0}}{\partial x \partial z}\right)\right)\right) d\tau = \hbar t\left(v_{z0}\frac{\partial w_x}{\partial z} + ikw_x - \frac{1}{\rho_0}\left(-ikp_0 + \eta(m^2 - k^2)v_{x0} + \eta\frac{\partial^2 w_x}{\partial z^2} + b_2(-k^2 v_{x0} - mikv_{z0})\right)\right) = \hbar t/\rho_0(\eta(k^2 - m^2)a_2 G_1 e^{ikx-mz} + b_2 G_1 e^{ikx-mz}(a_2 k^2 + ia_1 km) + G_1 ike^{ikx-mz} + \eta k_u/(z+z_m)^2) + \hbar t a_1 e^{ikx-mz}(k_u/(z+z_m) + 5/L) + \hbar t G_1 a_1 ike^{ikx-mz}\left(k_u \ln\left(\frac{z+z_m}{z_m}\right) + \frac{5z}{L}\right) \tag{59}$$

$$v_{z1} = \int_0^t \left(\frac{\partial v_{z0}}{\partial \tau} + w_x\frac{\partial v_{z0}}{\partial x} - \frac{1}{\rho_0}\left(-\frac{\partial p_{s0}}{\partial z} + \eta\left(\frac{\partial^2 v_{z0}}{\partial x^2} + \frac{\partial^2 v_{z0}}{\partial z^2}\right) + b_2\left(\frac{\partial^2 v_{z0}}{\partial z^2} + \frac{\partial^2 v_{x0}}{\partial x \partial z}\right)\right)\right) d\tau = -\hbar G_1 t/\rho_0 e^{ikx-mz}\left(m - a_1\eta k^2 + a_1\eta m^2 + a_1 b_2 m^2 - ia_2 b_2 km - ika_1\rho_0 k_u \ln\left(\frac{z+z_m}{z_m}\right)\right) - \hbar G_1 a_1 ikt5z/L e^{ikx-mz} \tag{60}$$

$$p_2(x,z,t) = p_1(x,z,t) + \hbar\int_0^t \left(\frac{\partial p_{s1}}{\partial \tau} + w_x\frac{\partial p_{s1}}{\partial x} + \rho_0 c^2\left(\frac{\partial v_{x1}}{\partial x} + \frac{\partial v_{z1}}{\partial z}\right)\right) d\tau = -\hbar(((c^2\rho(\hbar(G_1 k/\rho_0 e^{ikx-mz}(k - ia_2 k^2\eta - ia_2 b_2 k^2 + ia_2\eta m^2 + a_1 b_2 km) + G_1 a_2 k^2 e^{ikx-mz}\left(5z/L + k_u \log\left(\frac{z+z_m}{z_m}\right)\right) + G_1 a_1 ke^{ikx-mz}(5z + 5z_m + Lk_u)/(L(z+z_m))) - \hbar(G_1 a_1 5ik/L e^{ikx-mz} + (G_1 me^{ikx-mz} \cdot (m - a_1\eta k^2 + a_1\eta m^2 + \eta b_2 m^2 - ika_2 b_2 m - ika_1\rho_0 \log\left(\frac{z+z_m}{z_m}\right)))/\rho_0 + ikG_1 a_1 k_u e^{ikx-mz}/(z+z_m) - ikG_1 a_1 m5z/L)))/2 + Lk_u \cdot \log\left(\frac{z+z_m}{z_m}\right)))\left(ik5z + ikLk_u \log\left(\frac{z+z_m}{z_m}\right) + ikLa_2 c^2\rho_0 - La_1 c^2 m\rho_0\right)\left(-\frac{i}{2}\right))L^2)t^2 - G_1\hbar te^{ikx-mz}\left(ik5z/L + ikk_u \log\left(\frac{z+z_m}{z_m}\right) + ika_2 c^2\rho_0 - a_1 c^2 m\rho_0\right) + \left(G_1\hbar te^{ikx-mz}\left(ik5z/L + ikk_u \log\left(\frac{z+z_m}{z_m}\right) + ik\rho_0 a_2 c^2 - a_1 c^2 m\rho_0\right)\right) \tag{61}$$

按照类似的方法，可得到更高次的近似解，$p_m(x,z,t)(m = 3.4,\cdots)$. 那么，三阶近似解为

$$p_{3rd-order}(x,z,t) = p_0(x,z,t) + p_1(x,z,t) + p_2(x,z,t) \tag{62}$$

利用式(57)的水平风表达式，并引入有效声速，则有效声速，温度和风速的剖面如图 2 所示，该图对应于冬天的多云有风的一个夜晚[36].

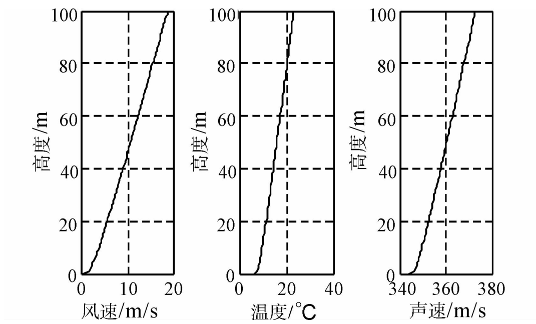


图 2　风速 $u(z)$、温度 $T(z)$、声速 $c(z)$ 剖面

Fig. 2　The wind speed, temperature and sound speed profile of a cloudy windy night

为了研究声波在有风有粘滞的大气中的传播特性及衰减特性，利用图 2 作为大气背景，对包含了风和粘滞特性的声压的三阶近似解进行了数值模拟，结果如图 3 所示.

由图 3 和图 4 可看出，随着传播距离的增大声压逐渐减小，且频率越高衰减越快，这与理论研究是相符的，因此风和粘滞特性是影响近地面声波传播特性的重要因素.

5　结　论

风和粘滞特性是影响声波传播特性的两个重要因素，本文首先由基本控制方程推导出了包含粘滞和风的耦合方程组，对匀质均一大气状态下的耦合方程组进行解析运算，得到了相速度和衰减的表达式，并在 MSISE90 大气模式下的物理背景对相速度和衰减系数进行了分析研究，之后利用 NMA 方法对该方程组精确解析解的求解，以此解析解作为初值，利用同伦分析方法对包含粘滞和风的耦合方程组进行了 3 阶近似解的求解，并以冬天的多云

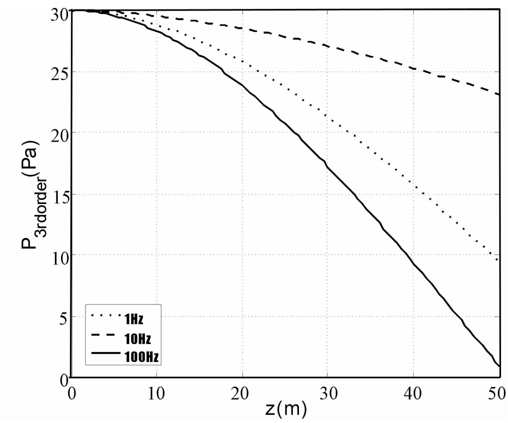


图3 不同频率下的三阶近似解声压特性

Fig. 3 3rd-order approximation sound pressure characteristics for different frequencies

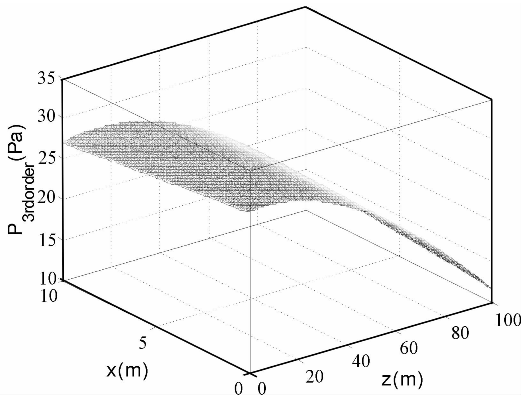


图4 三阶近似解声压的空间传播特性

Fig. 4 the spatial propagating characteristics of 3rd-order approximation sound pressure

有风的一个夜晚作为天气条件,进行了数值模拟,结果表明,随着传播距离的增加,声压峰值越小,且频率越大衰减越快,因此风和粘滞特性是影响近地面声波传播特性的重要因素.

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