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# 高压下 $\text{LaB}_6$ 的弹性和热力学性质的第一性原理计算

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**摘要:** 运用平面波赝势密度泛函理论, 研究了 CsCl 结构的  $\text{LaB}_6$  在高压下的弹性和热力学性质. 计算中使用了广义梯度近似, 得到在零温零压下  $\text{LaB}_6$  的晶格常数和已知的实验及其它理论结果相符. 同时, 我们还得到了  $\text{LaB}_6$  的弹性常数  $C_{ij}$ , 体弹模量  $B$ , 剪切模量  $G$ , 杨氏模量  $E$ , 德拜温度  $\Theta_E$ , 泊松系数  $\sigma$ , 压缩波速  $V_L$  和剪切波速  $V_S$  与压强的关系. 计算发现  $\text{LaB}_6$  在压强低于 14 GPa 时具有力学稳定性. 根据准谐德拜模型, 我们还预测了 CsCl 结构  $\text{LaB}_6$  的热力学性质, 对 0~14 GPa 和 0~1500 K 范围内热膨胀系数和比热容的变化进行了研究. 最后分析了  $\text{LaB}_6$  在零温零压和高压下的电子态密度图.

**关键词:** 密度泛函理论; 弹性性质; 电子性质; 热力学性质

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## Elastic and thermodynamic properties of $\text{LaB}_6$ under pressure: a first-principles study

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**Abstract:** The elastic and thermodynamic properties of CsCl-type structure  $\text{LaB}_6$  under high pressure are investigated by first-principles calculations based on plane-wave pseudopotential density functional theory method within the generalized gradient approximation (GGA). The calculated lattice parameters of  $\text{LaB}_6$  under zero pressure and zero temperature are in good agreement with the existing experimental data and other theoretical data. The pressure dependences of the elastic constants, bulk modulus  $B$  (GPa), shear modulus  $G$ , Young's modulus  $E$ , elastic Debye temperature  $\Theta_E$ , Poisson ratio  $\sigma$ , compressional wave velocity  $V_L$  and shear wave velocity  $V_S$  are also presented. An analysis for the calculated elastic constants has been made to reveal the mechanical stability of  $\text{LaB}_6$  up to 14 GPa. The thermodynamic properties of the CsCl-type structure  $\text{LaB}_6$  are predicted using the quasi-harmonic Debye model. The variations of thermal expansion coefficient  $\alpha$  and the specific heat capacity  $C_v$  are obtained systematically in the ranges of 0~14 GPa and 0~1500 K. At last, the pressure dependences of the density of states are also investigated.

**Keywords:** Density functional theory; Elastic properties; Electronic properties; Thermodynamic properties

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## 1 Introduction

The rare-earth hexaborides  $RB_6$  have attracted extensive experimental and theoretical interest due to their intriguing physical properties. For example,  $CeB_6$  is a dense Kondo compound and has interesting low-temperature magnetic phases<sup>[1]</sup>.  $SmB_6$  is also an exemplary Kondo insulator which features an energy gap in the electronic density of states (DOS) whose magnitude is strongly temperature dependent and only fully developed at low temperatures<sup>[2]</sup>.  $EuB_6$  is a ferromagnetic semiconductor with a transition temperature  $T_C = 15$  K. Below  $T_C$  the electrical resistivity is drastically reduced, above  $T_C$  a very large negative magnetoresistance is observed<sup>[3]</sup>. Among these compounds, lanthanum hexaboride ( $LaB_6$ ) has a special place.  $LaB_6$ , which is metallic at room temperature and becomes a superconductor at  $T_C = 0.45$  K<sup>[4, 5]</sup>, is a hard, refractory and stable material owing to strong B-B covalent bonds<sup>[6]</sup>. And it is a thermionic electron emitter with a low work function, high brightness and long life compared with conventional tungsten filaments<sup>[7]</sup>.

The structural, elastic, and thermodynamic properties of  $LaB_6$  have been investigated experimentally and theoretically by several groups. Early in 1977, Tanaka *et al.*<sup>[6]</sup> studied the elastic constants of  $LaB_6$  for the first time by the measurements of the transit time of pulses of longitudinal and transverse ultrasonic wave propagating in single crystal. An early electronic structure calculation was estimated by Kubo *et al.*<sup>[8]</sup>, using the three-dimensional Lock-Crisp-West (LCW) folded momentum densities (3D LCW FMD's) within local-density approximation (LDA) method. It's indicated that the Fermi surface topology plays an important role in the determination of structures. Mandrus *et al.*<sup>[9]</sup> explained the temperature dependence of the specific heat and resistivity of  $LaB_6$  well by using a model of La ions as independent Einstein oscillators embedded in a Debye framework of boron ions. Xu *et al.*<sup>[10]</sup> in-

vestigated the elastic and thermal properties of  $LaB_6$  in the framework of density-functional theory (DFT) with a quasi-harmonic Debye model. Bai *et al.*<sup>[11]</sup> achieved the structure and chemical bond characteristics of  $LaB_6$  by means of the density functional theory using the state-of-the-art full-potential linearized augmented plane wave (FPLAPW) method. In addition, Gürel *et al.*<sup>[12]</sup> performed an ab initio study of structural, elastic, lattice-dynamical, and thermodynamic properties of rare-earth hexaborides  $LaB_6$  within the density functional theory and linear-response formalism using pseudopotentials and a plane-wave basis. There have been many other works to investigate the  $LaB_6$  crystal and its properties<sup>[13, 14]</sup>.

What attracts us most is the pressure induced phase transition of  $LaB_6$ , which has recently provoked a great deal of controversy. By using the Raman and angle dispersive X-ray diffraction (ADXRD), Teredesai *et al.*<sup>[15]</sup> proposed that the pressure induced structural phase transition from CsCl-type structure to the orthorhombic structure occurs at around 10 GPa. While Godwal *et al.*<sup>[16]</sup> also using the Raman and ADXRD, proposed that there is no structural or electronic phase transition up to at least 25 GPa in CsCl-type structure.

The most common assessment of mechanical properties can be made by the determination of its elastic constants. Especially, the elastic constants of materials at high pressure are essential in order to predict and understand material response, strength, mechanical stability, and phase transition. The comprehensive analysis of elastic constants can provide a deeper insight into the hardness of materials. Furthermore, elastic properties are also related thermodynamically to the specific heat, thermal expansion, Debye temperature, melting point, and so on. Thus in this work, we put our investigation emphases mainly on the elastic and thermodynamic properties of CsCl-type structure  $LaB_6$  (space group  $Pm\bar{3}m$ ) under pressure. From the calculated elastic constants, we will study its mechanical stabilities and anisotropic behaviors, as well as the bulk modulus, shear

modulus, Young's modulus, Poisson's ratio, elastic Debye temperature of LaB<sub>6</sub> at diverse pressures. Because the mechanical properties of this substance are studied in detail for the first time, we hope that our work can provide useful help for future research in both experimental and theoretical studies. The rest of the paper is organized as follows.

The theoretical method is introduced and the computation details are given in Section 2. Some results and discussion are presented in Section 3. Finally, the summary of our main results and conclusions are given in Section 4.

## 2 Theoretical method and calculation details

### 2.1 Total energy electronic structure calculations

In the electronic structure calculations, we employ the plane-wave pseudopotential density functional theory method<sup>[17]</sup> through the Cambridge Serial Total Energy Package (CASTEP)<sup>[18]</sup> code together with both the generalized gradient approximation (GGA) proposed by Perdew *et al.*<sup>[19]</sup> and the local density approximation (LDA) proposed by Vosko *et al.*<sup>[20]</sup> for exchange-correlation potentials. A plane-wave basis set with energy cut-off 390 eV is applied. Pseudo-atom calculations are performed for La 5d<sup>1</sup>6s<sup>2</sup> and B 2s<sup>2</sup>2p. For the Brillouin-zone sampling, we use the 6 × 6 × 6 Monkhorst-Pack mesh<sup>[21]</sup>, where the self-consistent convergence of the total energy is at 5.0 × 10<sup>-7</sup> eV/atom. The tolerance for geometry optimization is set to within 5.0 × 10<sup>-6</sup> eV/atom, the maximum ionic force within 0.01 eV/Å, the maximum ionic displacement within 5.0 × 10<sup>-4</sup> Å, and the maximum stress within 0.02 GPa. The tolerance for elastic constants is set to within 1.0 × 10<sup>-6</sup> eV/atom, the maximum force within 0.0002 eV/Å, and the maximum strain amplitude within 0.003 GPa. These parameters are carefully tested. It is found that these parameters are sufficient to lead to a well-converged total energy.

### 2.2 Elastic properties

To calculate the elastic constants under hydrostatic pressure  $p$ , we use the symmetry-dependent strains that are non-volume conserving. The elastic constants,  $C_{ijkl}$ , with respect to the finite strain variables are defined as<sup>[22]</sup>

$$C_{ijkl} = \left( \frac{\partial \sigma_{ij}(x)}{\partial e_{kl}} \right)_x \quad (1)$$

where  $\sigma_{ij}$  and  $e_{kl}$  are the applied stress and Eulerian strain tensors, and  $X$ ,  $x$  are the coordinates before and after deformation, respectively. Under the hydrostatic pressure  $p$ , we have

$$C_{ijkl} = C_{ijkl} + \frac{p}{2} (2\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}) \quad (2)$$

where  $C_{ijkl}$  denote the second-order derivatives with respect to the infinitesimal strain (Eulerian),  $\delta$  is the finite strain variable. The fourth-rank tensor  $C$  generally greatly reduces when taking into account the symmetry of the crystal. In a cubic crystal, it is reduced to three components, *i. e.*  $C_{11}$ ,  $C_{12}$ , and  $C_{44}$ .

The bulk modulus  $B$  and the shear modulus  $G$  of the LaB<sub>6</sub> are taken as<sup>[23]</sup>

$$B = (B_R + B_V)/2 \quad (3)$$

$$G = (G_R + G_V)/2 \quad (4)$$

where R and V represent Reuss and Voigt boundaries, respectively.

$$B_V = B_R = (C_{11} + 2C_{12})/3 \quad (5)$$

$$G_V = (C_{11} - C_{12} + 3C_{44})/3 \quad (6)$$

$$G_R = \frac{5(C_{11} - C_{12})C_{44}}{[4C_{44} + 3(C_{11} - C_{12})]} \quad (7)$$

The polycrystalline Young's modulus  $E$  and the Poisson's ratio  $\sigma$  are then calculated from these elastic constants using the following relations<sup>[24]</sup>.

$$E = \frac{9BG}{3B + G} \quad (8)$$

$$\sigma = \frac{3B - 2G}{2(3B + G)} \quad (9)$$

The elastic Debye temperature  $\Theta_E$  may be estimated from the average sound velocity  $V_m$ <sup>[25]</sup>

$$\Theta_E = \frac{h}{k} \left[ \frac{3n}{4\pi} \left( \frac{N_A \rho}{M} \right) \right]^{1/3} V_m \quad (10)$$

where  $h$  is Planck's constants,  $k$  Boltzmann's

constant,  $N_A$  Avogadro's number,  $n$  the number of atoms per formula unit,  $M$  the molecular mass per formula unit,  $\rho$  the density, and  $V_m$  is obtained from<sup>[25]</sup>

$$V_m = \left[ \frac{1}{3} \left( \frac{2}{V_S^3} + \frac{1}{V_L^3} \right) \right]^{-1/3} \quad (11)$$

where  $V_S$  and  $V_L$  are the shear and longitudinal sound velocities, respectively. The probable values of the average shear and longitudinal sound velocities can be calculated by<sup>[26]</sup>

$$V_S = \sqrt{\frac{G}{\rho}}, V_L = \sqrt{\frac{B + (4/3)G}{\rho}} \quad (12)$$

### 2.3 Thermal properties

To investigate the thermal properties, we change the cell volume to obtain the corresponding energy, and then export them into the quasi-harmonic Debye model<sup>[27]</sup> to calculate the thermal properties. In this model, the non-equilibrium Gibbs function  $G(V; p, T)$  has the following from:

$$G^*(V; p, T) = E(V) + pV + A_{\text{vib}}(\Theta(V); T) \quad (13)$$

where  $E(V)$  is the total energy as a function of the cell volume  $V$ ,  $p$  is the hydrostatic pressure,  $\Theta(V)$  is the Debye temperature as a function of  $V$ , and  $A_{\text{vib}}$  is the vibrational Helmholtz free energy.

Based on this model, the specific heat  $C_v$ ,  $C_p$ , and the thermal expansion coefficient  $\alpha$  can be deduced from the following expressions:

$$C_v = 3n\kappa \left[ 4D(\Theta/T) - \frac{3\Theta/T}{e^{\Theta/T} - 1} \right] \quad (14)$$

$$\alpha = \frac{\gamma C_v}{B_T V} \quad (15)$$

where  $B_T$  is the static bulk modulus,  $\gamma$  is the Grüneisen parameter. They can be derived from  $B_T(p, T) = V \left( \frac{\partial^2 G^*(V; p, T)}{\partial^2 V} \right)_{p, T}$  and  $\gamma = -d \ln \Theta(V) / d \ln V$ , respectively.

Through the quasi-harmonic Debye model, one could obtain the thermodynamic quantities of  $\text{LaB}_6$  under pressure and high temperature. By applying the method, we have investigated the thermodynamic properties of several materials successfully<sup>[28-32]</sup>.

## 3 Results and discussion

### 3.1 Structural properties

To investigate the elastic and thermodynamic properties, we must determine the structures of  $\text{LaB}_6$  at first.  $\text{LaB}_6$  has a bcc-like structure (space group  $\text{Pm}\bar{3}\text{m}$ ) with La at the position  $(0, 0, 0)$  and B at the position  $(0.5, 0.5, x)$ , where  $x$  is the positional parameter of the B atoms. The structure information can be absolutely described by lattice parameter  $a$  and positional parameter  $x$ . To determine the ground state structure of  $\text{LaB}_6$ , we use the following steps. Firstly, we fix the lattice parameter  $a$  and take a series of different values of positional parameters  $x$  to calculate the total energies  $E$ , so that we can obtain an  $E-x$  curve and find a lowest energy  $E_{\text{min}}$ . The positional parameter  $x$  with the energy  $E_{\text{min}}$  is what we require. Secondly, with the obtained positional parameter  $x$ , we take a series of different values of lattice parameter  $a$  and repeat the above steps, the lattice parameter  $a$  also can be obtained. And for each  $a$ , we can calculate its corresponding primitive cell volume  $V$ , and then obtained the energy-volume ( $E-V$ ) curve of  $\text{LaB}_6$ .

By fitting the calculated  $E-V$  data to the third-order Birch-Murnaghan equation of state (EOS)<sup>[33]</sup>, the bulk modulus  $B_0$  at  $p=0$  and  $T=0$  can be obtained. All the equilibrium structure parameters and bulk moduli are listed in Tab. 1. It can be seen that our results of lattice parameter  $a$ , positional parameter  $x$  and bulk modulus  $B_0$  from GGA calculations are well consistent with the experimental data<sup>[34, 35]</sup> and other theoretical data<sup>[11, 14]</sup>. And the errors of lattice parameter  $a$  are less than 0.1%, respectively. On the other hand, our LDA results are not satisfactory, which are a little small when compared with the experimental<sup>[34]</sup> and other theoretical data<sup>[11]</sup>, except for bulk modulus  $B_0$ . Therefore, in this work, the GGA functional forms are applied in the following calculations.

Tab. 1 Calculated equilibrium lattice parameters of  $\text{LaB}_6$ , together with the experiment data and other theoretical results

	$a / \text{\AA}$	$x$	$B_0/\text{GPa}$	$B'_0$
Present GGA (PBESOL)	4.151	0.1995	181.15	3.68
Present LDA (CA-PZ)	4.140	0.1991	179.97	3.76
Cal. <sup>a</sup>	4.1277	0.1997	180	3.79
Cal. <sup>b</sup>	4.1605	0.2009	182.4	
Cal. <sup>c</sup>	4.1557	0.1998	173.6	3.72
Exp. <sup>d</sup>	4.1568		172.0	
Exp. <sup>e</sup>	4.1569	0.1996		

<sup>a</sup> Calculated through Linear-response theory within LDA<sup>[12]</sup>; <sup>b</sup> Calculated through PW-PP method within GGA(RPBE)<sup>[11]</sup>; <sup>c</sup> Calculated through PAW method within GGA(PBE)<sup>[14]</sup>; <sup>d</sup> Measured by X-Ray diffraction measurements<sup>[35]</sup>; <sup>e</sup> Measured by PLD and XRD measurements<sup>[34]</sup>.

The pressure and temperature dependence of the relative volume  $V/V_0$  of  $\text{LaB}_6$  are illustrated in Fig. 1. It is shown that, as the applied pressure increases from 0 to 14 GPa, the volume of  $\text{LaB}_6$  decrease linear at the giving temperature, and the relative volume  $V/V_0$  of higher temperature is less than that of lower temperature at the same pressure. This means, under higher temperature,  $\text{LaB}_6$  is easier to be compressed, as temperature could make  $\text{LaB}_6$  soft.

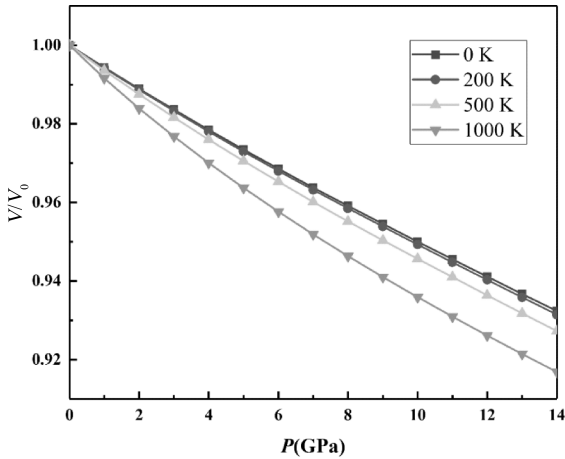


Fig. 1 Normalized primitive unit cell volume  $V/V_0$  as a function of pressure

### 3.2 Elastic properties

We list our calculated elastic constants and

aggregate elastic modulus  $B$  of the cubic structure  $\text{LaB}_6$  at 0 K and 0 GPa in Tab. 2. It can be seen clearly that our results are in agreement with the other theoretical<sup>[12, 14]</sup> and experimental data<sup>[36]</sup>, which indicates that our results are reasonable. In addition, from the Eq. (3), the bulk modulus  $B$  (in Tab. 2) obtained by our elastic constants is 177.9 GPa, which is consistent with the value estimated by fitting the  $E - V$  data mentioned above.

Tab. 2 Calculated elastic constants  $C_{ij}$  of  $\text{LaB}_6$  at 0 K and 0 GPa, in comparison with the experimental data and other theoretical results

	$C_{11}/\text{GPa}$	$C_{44}/\text{GPa}$	$C_{12}/\text{GPa}$	$B/\text{GPa}$
Present	460.6	94.0	36.6	177.9
Cal. <sup>a</sup>	466	88	37	180
Cal. <sup>b</sup>	473	92	24	173.6
Exp. <sup>c</sup>	463	89	45	184

<sup>a</sup> Calculated through Linear-response theory within LDA<sup>[12]</sup>; <sup>b</sup> Calculated through PAW method within GGA(PBE)<sup>[14]</sup>; <sup>c</sup> Measured by X-Ray diffraction measurements<sup>[36]</sup>.

The pressure dependences of the elastic constants ( $C_{11}$ ,  $C_{12}$  and  $C_{44}$ ) of  $\text{LaB}_6$  under pressure up to 14 GPa are summarized in Tab. 3. Unfortunately, to our knowledge, no experimental and theoretical data of the elastic constants are available to compare with our results in high pressure. The elastic constant  $C_{11}$  represents the elasticity in length, while  $C_{12}$  and  $C_{44}$  are related to the elasticity in shape. We find that all three elastic constants increase almost linearly with the increasing pressure which also can be observed from Fig. 2. Among them,  $C_{11}$  is more sensitive to pressure than  $C_{12}$  and  $C_{44}$ . For a cubic crystal, the mechanical stability leads to restrictions on the elastic constants under isotropic pressure as follows<sup>[37]</sup>:  $\tilde{C}_{44} > 0, \tilde{C}_{11} > |\tilde{C}_{12}|, \tilde{C}_{11} + 2\tilde{C}_{12} > 0$ , where  $\tilde{C}_{ii} = C_{ii} - p$  ( $i = 1, 4$ ),  $\tilde{C}_{12} = C_{12} + p$ . It is obvious from Tab. 3 that the elastic constants of  $\text{LaB}_6$  satisfy all of these conditions at pressure up to 14 GPa. It is known that being a fourth-rank tensor property, elasticity is anisotropic for a cubic crystal, and it is conveniently expressed by the dimensionless parameter  $A = 2C_{44}/(C_{11} - C_{12})$ . For

isotropic elasticity, double  $C_{44}$  are equal to  $(C_{11} - C_{12})$ , thus  $A=1$ . Through the calculated elastic constants, one can obtain the Zener's anisotropy parameter  $A$  at different pressures and 0 K, which is also presented in Tab. 3. Our calculated  $A$  being 0.443 at 0 GPa and 0 K is in agreement with the value obtained by Duan *et al.* [14] ( $A=0.42$  at 0 GPa and 0 K). It is showed that the Zener's anisotropy parameter  $A$  almost doesn't change with the elevated pressure  $p$  at 0 K, indicating that  $A$  deviates from 1 and the anisotropy of the cubic structure  $\text{LaB}_6$  almost keeps stable in the processes of increasing pressure.

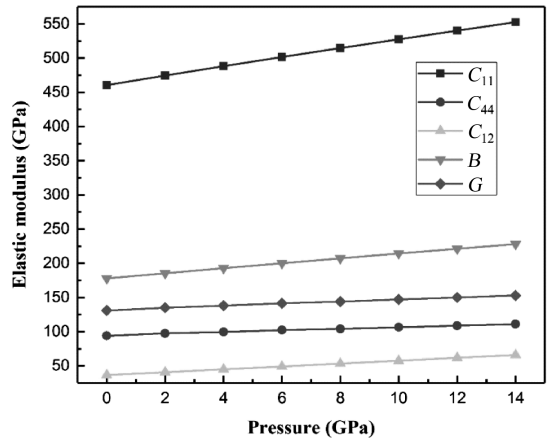


Fig. 2 Pressure dependencies of elastic constants of  $\text{LaB}_6$  at 0 K

Tab. 3 Calculated elastic constants  $C_{ij}$  (GPa), bulk modulus  $B$  (GPa), shear modulus  $G$  (GPa),  $B/G$ , acoustic velocities, and  $V_L$  and  $V_S$  (km/s), and elastic Debye temperature  $\Theta_E$  (K) of  $\text{LaB}_6$  under pressure  $p$  (GPa)

$p$	0	2	4	6	8	10	12	14
$C_{11}$	460.6	474.4	488.3	501.6	514.8	527.6	540.2	552.6
$C_{44}$	94.0	97.5	99.6	102.3	104.1	106.4	108.8	111.0
$C_{12}$	36.6	40.7	45.0	49.2	53.4	57.6	61.7	65.9
$B$	177.9	185.3	192.8	200.0	207.2	214.3	221.2	228.1
$G$	131.0	135.1	138.1	141.4	144.1	147.0	150.0	152.9
$B/G$	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49
$E$	315.6	326.1	334.4	343.3	350.9	358.9	367.1	374.9
$\sigma$	0.204	0.207	0.211	0.214	0.218	0.221	0.223	0.226
$V_L$	8.63	8.75	8.84	8.93	9.01	9.09	9.17	9.24
$V_S$	5.26	5.32	5.35	5.39	5.41	5.44	5.47	5.50
$\Theta_E$	898	911	920	929	937	945	954	962
$A$	0.443	0.449	0.449	0.452	0.451	0.452	0.454	0.456

In Tab. 3, we also list the bulk modulus and shear modulus, which can easily describe the hardness of a crystal in an indirect way. It is found that both bulk modulus  $B$  and shear modulus  $G$  increase gradually with the increasing pressure. This implies that the compressibility of  $\text{LaB}_6$  becomes lower as the pressure increases. From the ratio of  $B/G$ , one can distinguish the ductility and brittleness of metals. The threshold is around 1.75 [38]. When  $B/G > 1.75$ , the material behaves in a ductile manner, otherwise the material behaves in a brittle manner. Duan *et al.* [14] obtained the  $B/G$  is 1.33 at 0 GPa and 0 K. The  $B/G$  as a function of pressure is displayed in Tab. 3. It can be seen that the value of  $B/G$  increases with the increasing pressure, indicating

that it becomes much harder with the increasing pressure, and it is brittle in nature up to 14 GPa.

Young's modulus is defined as the ratio of stress to strain, and is used to provide a measure of the stiffness of the solid, *i. e.* the larger the value of  $E$ , the stiffer the material. Tab. 3 illustrates that Young's modulus increases with pressure when  $p < 14$  GPa, indicating that the pressure can, to some extent, improve the stiffness of this material. Poisson's ratio is defined as the absolute value of ratio of transverse strain to longitudinal strain, when materials subject to longitudinal stress. Poisson's ratio  $\sigma = 0.25$  is the lower limit for central force solids and 0.5 is the upper limit. The previous work obtained the Poisson ratio is 0.19 at 0 GPa and 0 K [14], which is in a

agreement with our work. Tab. 3 shows us that the Poisson's ratio of LaB<sub>6</sub> increases from 0.204 to 0.226 with the pressure up to 14 GPa. This means the interatomic forces in LaB<sub>6</sub> is now-central forces.

According to the elastic constants obtained, we can also obtain the compressional and shear wave velocities of LaB<sub>6</sub> under pressure. We list them in Tab. 3, and the results of them are  $V_L = 8.63$  km/s and  $V_S = 5.26$  km/s at 0 GPa, which is also in agreement with previous study ( $V_L = 8.625$  km/s and  $V_S = 5.306$  km/s at 0 GPa, 0 K)<sup>[14]</sup>. It is shown that in Fig. 3 that with the increasing pressure, the shear wave velocities increase. The compressional wave velocities change slowly with the elevated pressure. Unfortunately, there are no experimental data or theoretical data to be compared with our results.

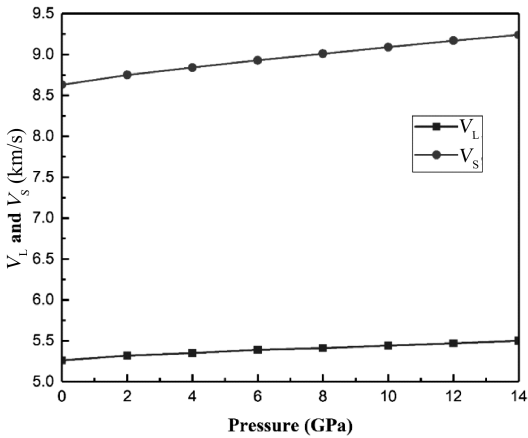


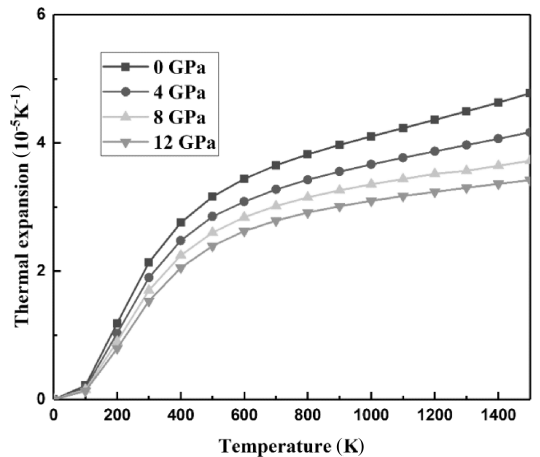
Fig. 3 The compressional and shear wave velocities of LaB<sub>6</sub> as a function of pressure at  $T=0$  K

As is known, the Debye temperature is an important fundamental parameter and closely related to many physical properties of solids, such as the specific heat and melting temperature. From the elastic constants, one can obtain the elastic Debye temperature  $\Theta_E^{[25, 26]}$ . The obtained elastic Debye temperatures of LaB<sub>6</sub> under pressure are also presented in Tab. 3. For LaB<sub>6</sub> at 0 K and 0 GPa, we yield 898 K from the elastic constants of GGA calculations, which has discrepancy comparing with 1165 K obtained through quasi-harmonic Debye model at 0 K and 0 GPa by Xu *et al.*<sup>[10]</sup>, but is consistent with the experiment val-

ue 878 K through ultrasonics and XRD at room temperature by Petropoulos *et al.*<sup>[39]</sup>. Obviously, the Debye temperature increases monotonically with increasing pressure up to 14 GPa.

### 3.3 Thermodynamic properties

The thermal expansion coefficient and specific heats  $C_v$  are the important reference to predict material properties, especially for the thermodynamic properties. We present the variations of the thermal expansion  $\alpha$  and specific heats  $C_v$  with temperature and pressure in Figs. 4 and 5 respectively. Our calculated value for  $\alpha$  is equal to  $2.14 \times 10^{-5} \text{ K}^{-1}$  at 300 K, which is in agreement with the value obtained by Chen *et al.* ( $\alpha = 2.10 \times 10^{-5} \text{ K}^{-1}$  at 298 K)<sup>[34]</sup> and Xu *et al.* ( $\alpha = 2.11 \times 10^{-5} \text{ K}^{-1}$  at 298 K)<sup>[10]</sup>. Seen from the Fig. 4, at a given pressure,  $\alpha$  increases exponentially at low temperatures and gradually approaches a linear increase at high temperatures. As the pressure increases, the growth trend of  $\alpha$  with temperature becomes smaller and smaller, especially at high temperatures. However, at a given temperature,  $\alpha$  decreases drastically with the increasing pressure. When the pressure increases to above 10 GPa, the thermal expansion  $\alpha$  of 900 K is just a little larger than that of 600 K, and the curves of 600, 900, and 1200 K seem to be consistent at high pressure, which means that the temperature dependence of  $\alpha$  is very small at high pressures and high temperatures.



(a)

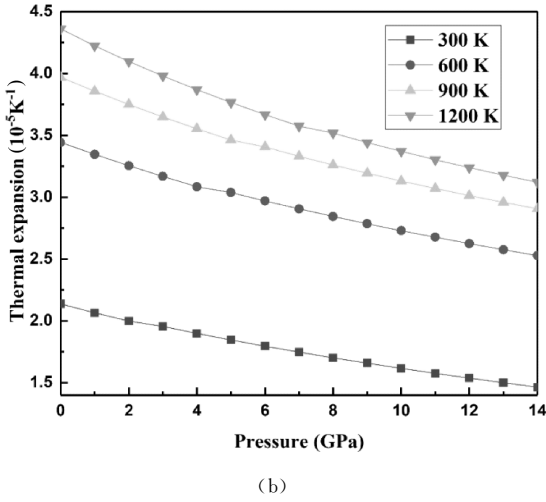


Fig. 4 Thermal expansion versus temperature and pressure of  $\text{LaB}_6$

In Fig. 5, we can find that the  $C_v$  rise rapidly with the temperature at low temperatures, but at high temperatures, the anharmonic effect on  $C_v$  is suppressed,  $C_v$  converges slowly to the Dulong-Petit limit, about  $173 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$ , is consistent with the theoretical value  $175 \text{ J} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$  by Xu *et al.*<sup>[10]</sup> It also shows that the effect of temperature on the special heat capacity  $C_v$  is much larger than that of pressure.

In Fig. 6, we plotted the temperature dependence of the Debye temperature. It is well known that the Debye temperature is proportional to the bulk modulus and that hard materials exhibit elevated Debye temperatures. Our calculated Debye temperature  $\Theta_D$  equals 1100 K at 300 K, in good agreement with the  $\Theta_D$  calculated by Xu *et al.* ( $\Theta_D = 1161.5 \text{ K}$  at 300 K)<sup>[10]</sup>. From Fig. 6, it is clearly that when  $T < 200 \text{ K}$ ,  $\Theta_D$  remains nearly constant. And then,  $\Theta_D$  decreases as the temperature increases, when  $T > 400 \text{ K}$ , the variation of  $\Theta_D$  with temperature is almost linearly increased. We can find the Debye temperature keeps above 1010 K when the temperature is up to 1500 K, which means that the effect of temperature on Debye temperature is moderate. We note the difference between the elastic Debye temperature  $\Theta_E$  calculated by elastic constants (in Tab. 3) and the Debye temperature  $\Theta_D$  estimated by thermodynamic methods. The elastic Debye temperature  $\Theta_E$  and the Debye temperature  $\Theta_D$  are 898 and

1110 K at 0 K and 0 GPa. This difference also appears in some other materials<sup>[1, 40]</sup>. This is partially because the Debye temperature  $\Theta_D$  is obtained under the assumption that the material is isotropic, but  $\text{LaB}_6$  is anisotropic. These results are consistent with the elastic anisotropic parameter  $A$ .

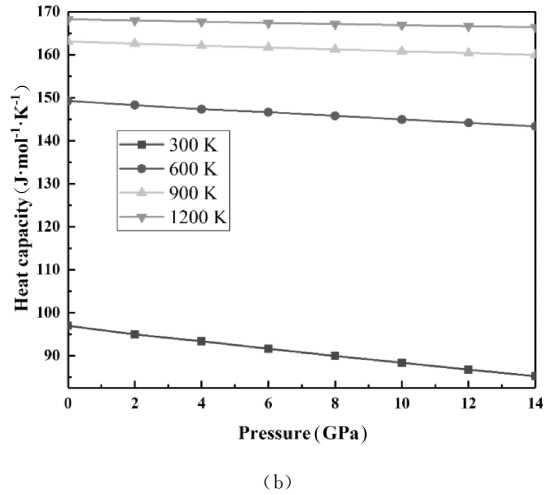
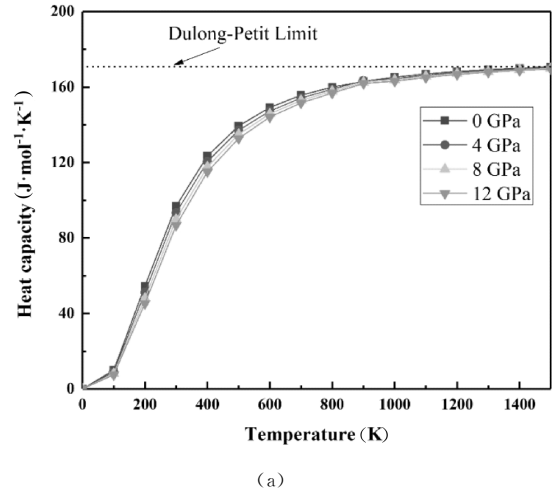


Fig. 5 The heat capacity  $C_v$  of  $\text{LaB}_6$  as a function of temperature  $T$  at several pressures

### 3.4 Electronic structure

The density of states (DOS) plays an important role in the analysis of the physical properties of materials. The calculated partial density of states (PDOS) for  $\text{LaB}_6$  is shown in Fig. 7. Our calculations at 0 GPa are consonant with the results obtained by Hossian *et al.*<sup>[13]</sup>. It is clear from the figure that the valence band can be divided into two parts. The isolated part is around



$-15.0$  eV with a narrow width, which arises mainly from an equal contribution of both B 2s and 2p states. The main part of valence band has a width of about 9 eV, and consists mainly of the B sp states and slightly of the La d state which is completely occupied and a small contribution of the La 6s states. It can be divided into two parts, the first part is located in the range of  $-11$  to  $-7.5$  eV (part I); the second one is between  $-7.5$  and  $-1$  eV (part II), and the bands located at around  $-10$  and  $-5$  eV have a remarkably localized characteristic. The part I originates from almost equal contributions of B 2s and 2p orbitals, while the part II is dominated by the B 2p orbital weakly hybridized with La 5d orbitals. Around the Fermi level, it is clearly that the B 2p state shows a strong hybridization with the La 5d states. Similarly, the conduction band can be divided into two parts depending on the weight of the PDOS, the first part is located in the range of 1 to 6 eV (part III); the second one is between 6 and 11 eV (part IV). In part III, the band has a remarkably localized characteristic at 3 eV and it is dominated by La 5d hybridized with the B 2sp states. While the part IV is populated with all the states of both B and La atoms.

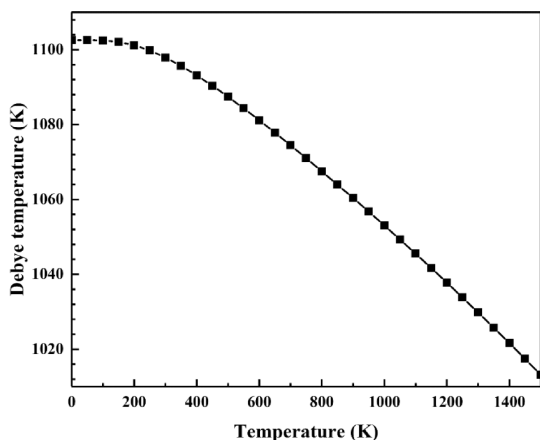


Fig. 6 Debye temperature versus temperature for  $\text{LaB}_6$  at zero pressure

The calculated total density of states (TDOS) and partial density of states (PDOS) at high pressure (14 GPa) are also illustrated in Fig. 7. It is found that: (1) the TDOS values become larger at the sharp  $-5$  eV when the pressure is increased, as well

as the peak value of B-2s, thus, we can predict that the obtained peak is due to 2s of B; (2) all the DOS peak moves left slowly.

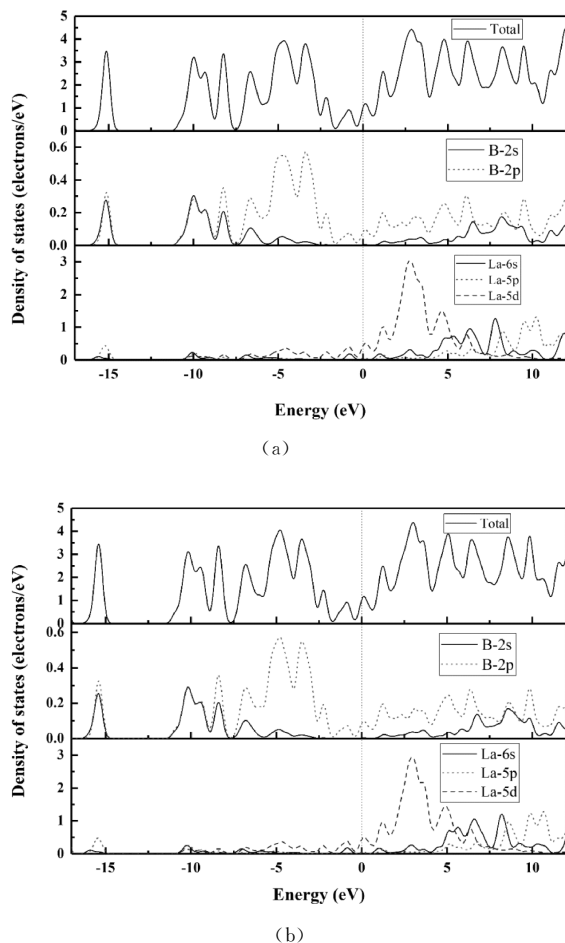


Fig. 7 Calculated total and partial density of states of structure  $\text{LaB}_6$  at 0 GPa (a) and 14 GPa (b)

## 4 Conclusions

We have investigated the electronic structure and elastic properties of  $\text{LaB}_6$  under pressure in the frame of the density functional theory. The calculated lattice parameters of  $\text{LaB}_6$  at zero pressure and zero temperature from GGA are in agreement with the available experimental and theoretical data. The pressure dependences of elastic parameters (including elastic constants, bulk modulus, shear modulus, Young's modulus and Poisson's ratio) are also obtained. We have found that they all increase linearly with the increasing pressure. With these elastic parameters, we have studied the mechanical properties of  $\text{LaB}_6$  under

pressure, and predicted that they are stable under pressure up to 14 GPa. We also have investigated the thermal properties of  $\text{LaB}_6$  by utilizing quasi-harmonic Debye model. The thermal calculations show that the thermal expansion coefficient is positively related to the temperature, and negatively related to the pressure. And the effect of temperature on the special heat capacity  $C_v$  is much larger than that of pressure. Finally, we investigate the variety of the total density of states and the partial density of states of  $\text{LaB}_6$  at diverse pressures.

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