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加权 Banach 空间上的复合算子的不交超循环性

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摘 要：本文研究了加权 Banach 空间 $H_{\alpha,0}^{\infty}$ 上的复合算子的不交超循环性. 根据解析映射的不同，本文给出了判断复合算子具有不交超循环性和不交亚超循环性的充分条件. 此外，本文还刻画了该空间上的加权复合算子的超循环性.

关键词：不交超循环性；复合算子；加权 Banach 空间
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Disjoint hypercyclicity of composition operators on weighted Banach space of holomorphic functions

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Abstract: In this paper, we discuss disjoint hypercyclicity of the composition operators on the weighted Banach space $H_{\alpha,0}^{\infty}$. Underlying the difference of the analytic maps, we obtain some sufficient conditions for the disjoint hypercyclicity and disjoint supercyclicity of composition operators. We also obtain a partial characterization for the hypercyclicity of weighted composition operators on.

Keywords: Disjoint hypercyclic; Composition operator; Weighted Banach space
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1 Introduction

Let D be the open unit disk in the complex place $\mathbf{C}$, $H(D)$ and $S(D)$ denote the set of all holomorphic functions on D and the collection of all holomorphic self maps of D. Each $u \in H(D)$ and $\varphi \in S(D)$ induce a linear weighted composition operator $uC_{\varphi}: H(D) \to H(D)$ by $uC_{\varphi}(f)(z) = u(z)f(\varphi(z))$, for every $f \in H(D)$ and $z \in D$. When $u = 1$ and $\varphi \in S(D)$, we obtain the composition operator C_{φ}. For more information about composition operators, we refer the readers to see Refs. [1~3].

Let $B(X)$ be the spaces of bounded linear operator on a separable infinite dimensional Banach space X. $T \in B(X)$ is called hypercyclicity if there exists a vector $x \in B(X)$ such that the orbit $orb(T, x) = \{T^n(x): n \in \mathbf{N}\}$. Such a vector x is said to be hypercyclic for T. We refer the readers to Ref. [4] for more examples and background about linear dynamics.

In this paper, we consider the hypercyclicity of multiples of composition operators on $H_{\alpha,0}^{\infty}$, which has been characterized in Ref. [5], where

$$H_{\alpha}^{\infty} = \{f \in H(D), \| f \|_{\alpha} = \sup_{z\in D}(1 - | z |^2)^{\alpha} | f(z) | \},$$

$$H_{\alpha,0}^{\infty} = \{f \in H_{\alpha}^{\infty}, \| f \|_{\alpha} =$$

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$\lim_{|z|\to 1}(1-|z|^2)^\alpha |f(z)|\}$.

A map φ is called to be a linear fractional map $\varphi(z)=\frac{az+b}{cz+d}$ where $ab-cd\neq 0$. If $\varphi(D)\subset D$, φ is said to be a linear fractional self-map of D. We write $LFT(D)$ to refer to the set of all such map.

Theorem 1.1 (Denjoy-Wolff) If $\varphi: D\to D$ is analytic map with no fixed point in D, then there exists a point $w\in\partial D$ such that $\varphi^n\to w$ uniformly on compact subsets of D.

We call φ to be elliptic type if $\varphi\in \mathrm{Aut}(D)$, and φ has a fixed point in D. By Dejoy-Wolff iteration theorem, we know the nonelliptic type can be divided into three parts. Let w be the Denjoy-Wolff point of φ, we say φ is of dilation type if $w\in D$ hyperbolic type if $w\in\partial D$ and $\varphi'(w)<1$, and parabolic type if $w\in\partial D$ and $\varphi'(w)=1$. Miralles and Wolf[6] studied the hypercyclic acting on $H_v^0(D)$ induced by linear fractional maps.

Motivated by the above results, we characterize disjoint hypercyclicity and disjoint supercyclicity of composition operators on $H_{\alpha,0}^\infty$. We construct different dense subsets $X_0, X_1, \cdots, X_N$ mappings $S_{l,k}: X_l\to X\ (1\leqslant l\leqslant N, k\in\mathbf{N})$ and some proof skills completely different from the skills in Hardy space and $H_{v,0}$. More specifically, we use the Denjoy-Wolff Theorem and the fact that the multiplication operator M_{z-w} has dense range in $H_{\alpha,0}^\infty$ to construct the above dense subsets. Then we obtain the mappings $S_{l,k}$[7] which satisfy the second condition of d-Hypercyclicity Criterion[8]. Besides, by the definition of norm on $H_{\alpha,0}^\infty$ and the weight $v(z)=(1-|z|^2)^\alpha$, we get the third condition of d-Hypercyclicity Criterion. In general, the definitions of norm are different in the Hardy space and $H_{\alpha,0}^\infty$. For the special composition operators, we obtain some sufficient conditions of the disjoint hypercyclicity and disjoint supercyclicity on $H_{\alpha,0}^\infty$. Finally, we also obtain a partial characterization of the hypercyclicity of weighted composition operators on $H_{\alpha,0}^\infty$.

2 Main results

In this section we discuss disjoint hypercyclicity and disjoint mixing behaviour of finitely many linear fractional composition operators on $H_{\alpha,0}^\infty$.

Definition 2.1[9,10] The operators $(T_{1,n})_{n=1}^\infty, \cdots, (T_{N,n})_{n=1}^\infty$ acting on the same topological vector space X are disjoint hypercyclic (respectively, disjoint supercylic) or d-hypercyclic (respectively, d-supercyclic) for short if there exists a vector in $x\in X$ such that the vector $(x,\cdots,x)$ is hypercyclic vector (respectively, supercyclic vector) for the direct sum operator $T_1\oplus T_2\oplus\cdots\oplus T_N$ on the product space $\bigoplus_{i=1}^N X$. Any such vector is called a d-hypercyclic vector (respectively, d-supercyclic vector) for the operators $T_1, \cdots, T_N$. If the d-hypercyclic vectors (respectively, d-supercyclic vectors) form a dense set in X, we call the operators $T_1, \cdots, T_N$ d-hypercyclic (respectively, d-supercyclic).

Definition 2.2 We say that $N\geqslant 2$ sequences of operators $(T_{1,n})_{n=1}^\infty, \cdots, (T_{N,n})_{n=1}^\infty$ in $B(X)$ are d-topologically transitive (respectively, d-mixing), provided for any non-empty open subset $V_0, \cdots, V_N$ there exists $m\in\mathbf{N}$ such that

$$V_0\cap T_{1,m}^{-1}(V_1)\cap\cdots\cap T_{N,m}^{-1}(V_N)\neq\varnothing$$

(respectively, $V_0\cap T_{1,m}^{-1}(V_1)\cap\cdots\cap T_{N,m}^{-1}(V_N)\neq\varnothing$ for all $j\geqslant m$). Also, we say that $N\geqslant 2$ operators $T_1, \cdots, T_N$ in $B(X)$ are d-topologically transitive (respectively, d-mixing), provided that $(T_{1,n})_{n=1}^\infty, \cdots, (T_{N,n})_{n=1}^\infty$ are d-topologically transitive sequences (respectively, d-mixing sequences).

The following definition so-called d-Hypercyclicity Criterion[8].

Definition 2.3 Let (n_k) be a strictly increasing sequence of positive integers. We say that $T_1, \cdots, T_N$ satisfy the d-Hypercyclicity Criterion with respect to (n_k), provided there exist dense subsets $X_0, X_1, \cdots, X_N$, of X and mappings $S_{l,k}: X_l\to X (1\leqslant l\leqslant N, k\in\mathbf{N})$ satisfying

(1) $T_l^{n_k}\to 0$ pointwise on X_0;

(2) $S_{l,k}\to 0$ pointwise on X_l;

(3) $T_l^{n_k}S_{l,k}\to Id_{i,l}X_l$ pointwise on X_l.

In general, we say that $T_1, \cdots, T_N\in L(X)$ satisfy the d-Hypercyclicity Criterion if there exist some sequence n_k, for which the above conditions are satisfied.

Lemma 2.4 For $\alpha>0$, $w\in\mathbf{C}\backslash D$, the multiplication operator $M_{z-w}:H_{\alpha,0}^{\infty}\rightarrow H_{\alpha,0}^{\infty}$,

$$f(z)\rightarrow(z-w)f(z).$$

Proof Since the set of all polynomial $p\in H_{\alpha,0}^{\infty}$ by range of function M_{z-w}. Without loss of generality, we assume that $w\in\mathbf{R}$ and $p(w)=a$. Since $\lim\limits_{|z|\rightarrow1}(1-|z|^2)^{\alpha}=0$, for any $\varepsilon>0$, there exist $0<r<1$ and $n_0\in\mathbf{N}$ such that

$$(1-|z|^2)^{\alpha}<\varepsilon\frac{w_0^n}{|a|}$$

if

$$|z|>r \text{ and } r^{n_0}<\varepsilon\frac{w_0^n}{|a|}.$$

Then the polynomials $q(z)=p(z)-\frac{z^{n_0}}{w^{n_0}}a$. It is obvious that $q(w)=0$ and $q(z)\in H_{\alpha,0}^{\infty}$, that is, $q\in$ range of M_{z-w}. It follows that

$$\sup_{z\leqslant r}(1-|z|^2)^{\alpha}|p(z)-q(z)|=$$

$$\sup_{z\leqslant r}(1-|z|^2)^{\alpha}\left|\frac{z^{n_0}}{w^{n_0}}a\right|\leqslant\frac{r^{n_0}}{w^{n_0}}|a|<\varepsilon,$$

$$\sup_{z>r}(1-|z|^2)^{\alpha}|p(z)-q(z)|=$$

$$\sup_{z>r}(1-|z|^2)^{\alpha}\left|\frac{z^{n_0}}{w^{n_0}}a\right|\leqslant\frac{|a|}{w^{n_0}}|a|<\varepsilon.$$

So

$$\|p(z)-q(z)\|_{\alpha}=$$

$$\sup_{z\in D}(1-|z|^2)^{\alpha}|p(z)-q(z)|<\varepsilon.$$

This completes the proof.

Corollary 2.5[11] Let $\alpha>0$, m a positive integer, and $A\subset\mathbf{C}\backslash D$ be a finite set. Then the subspace of polynomials vanishing at each point of A dense in $H_{\alpha,0}^{\infty}$.

Lemma 2.6 Suppose that $\varphi:D\rightarrow D$ is an holomorphism function and φ has no fixed point in D. If $w\in\mathbf{C}\backslash D$ is the Dejoy-Wolff point of φ, then for any holomorphic function p,

$$\lim_{k\rightarrow\infty}\|p(\varphi^k)-p(w)\|_{\alpha}=0.$$

Proof Set $M=\max\{\max\limits_{z\in\overline{D}}\{|p(z)|,(1-|z|^2)^{\alpha}\},|p(w)|\}$. Since p is bounded holomorphic function, for any $\varepsilon>0$, there exists $\delta>0$, such that

$$|p(z)-q(w)|<\frac{\varepsilon}{2M}$$

when $|z-w|<\delta$. Since $\lim\limits_{|z|\rightarrow1}(1-|z|^2)^{\alpha}=0$, there exists $0<r<1$, such that

$$(1-|z|^2)^{\alpha}<\frac{\varepsilon}{4M},\ \forall z\in D \text{ and } |z|>r.$$

Because w is the Denjoy-Wolff point of φ, there exists $k_0\in\mathbf{N}$, such that

$$|\varphi^k(z)-w|<\delta,\ \forall k\geqslant k_0.$$

Thus

$$|p(\varphi^k(z))-p(w)|<\frac{\varepsilon}{2M},\ \forall k\geqslant k_0.$$

So, it follows that

$$\sup_{z\in D}(1-|z|^2)^{\alpha}|p(\varphi^k)-p(w)|\leqslant$$

$$\max\{\sup_{z\leqslant r}(1-|z|^2)^{\alpha}|p(\varphi^k)-p(w)|,$$

$$\sup_{z>r}(1-|z|^2)^{\alpha}|p(\varphi^k)-p(w)|\}\leqslant$$

$$\frac{\varepsilon}{2M}M+\frac{\varepsilon}{4M}2M=\varepsilon$$

for any $k\geqslant k_0$. This completes the proof.

Lemma 2.7[7] Let $\varphi_1,\varphi_2\in LET(D)$ be distinct. Suppose that for $i=1,2$, the map $\varphi_i\in LET(D)$ is either parabolic or hyperbolic, β_i and γ_i are attractive fixed points of φ_i and φ_i^{-1}, respectively (so φ_i is parabolic if and only if $\beta_i=\gamma_i$). Suppose that it can not happen that both $\beta_1=\beta_2$ and $\varphi_1'(\beta_1)=\varphi_2'(\beta_2)<1$ hold. Then

$$\varphi_1^{-[n]}\circ\varphi_2^{[n]}\rightarrow\mu_1,n\rightarrow\infty$$

and

$$\varphi_2^{-[n]}\circ\varphi_1^{[n]}\rightarrow\mu_2,n\rightarrow\infty$$

locally and uniformly on $\mathbf{C}\backslash\{\beta_1,\gamma_1,\beta_2,\gamma_2\}$, where $\mu_i\in\{\beta_i,\gamma_i\}$, $i=1,2$.

Proposition 2.8 Let $T_1,\cdots,T_N\in L(X)$ satisfy the Disjoint Hypercyclicity Criterion with respect to a sequence (n_k). Then the sequences $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$ are d-mixing. In particular, $T_1,\cdots,T_N\in L(X)$ are d-hypercyclic.

Theorem 2.9 Let $N\geqslant2$, $\alpha>0$ and $1<p<\infty$. For each $i=1,2,\cdots N$, let $\varphi_i\in LET(D)$ be either a parabolic automorphism or a hyperbolic map without fixed points in D. Suppose that there are no two symbols φ_i,φ_j having the same attractive fixed point β such that $\varphi_i^{-1}(\beta)=\varphi_j^{-1}(\beta)<1$. Then the operators $C_{\varphi_1},\cdots,C_{\varphi_N}$ are d-hypercyclic on $H_{\alpha,0}^{\infty}$.

proof For $1\leqslant i\leqslant N$, let $\beta_i,\gamma_i\in\widetilde{\mathbf{C}}$ denote the attractive fixed points of φ_i and φ_i^{-1}, respective(so φ_i, is parabolic if and only if $\beta_i=\gamma_i$). Without loss of generality, we suppose that $\beta_i\in\partial D$ and $\gamma_i\in\mathbf{C}\backslash D$ for each $1\leqslant i\leqslant N$. Let X_0 denote the set of polynomials vanishing at every fixed point of $C_{\varphi_1},\cdots,C_{\varphi_N}$. By Corollary 2.5, the set X_0 is dense in $H^{\infty}_{\alpha,0}$. By Denjoy-Wolff Theorem, we obtain that $\varphi_1^{[k]}\to w$, when $k\to\infty$ on any subsets of D. Thus $f\circ\varphi_1^{[k]}\to f(w)=0$, when $k\to\infty$,

$$\|C^k_{\varphi_l}f\|_\alpha=\|f\circ\varphi_l^{[k]}\|_\alpha\to\|f(w)\|_\alpha=0$$

for every $f\in X_0$. That is, $C^k_{\varphi_l}\to 0$ pointwise on X_0, when $k\to\infty(1\leqslant l\leqslant N)$.

Next, for each $1\leqslant i\leqslant N$, let $X_i=\{f\circ\sigma_i: f\in P_B\}$, where P_B denote the set of polynomials vanishing on $A=\{\beta_i,\sigma_i(\gamma_i)\}$, where $\sigma_i\in\mathrm{Aut}(D)$ is defined as follows: If $\varphi_i\in\mathrm{Aut}(D)$ denote the identity map on $\mathbf{C}$, else, pick $\sigma_i\in\mathrm{Aut}(D)$ satisfying $\sigma_i(\beta_i)=\beta_i$, and $\sigma_i((1+\frac{2}{3})\beta_i)=\infty$ where $0<\sigma<|\beta_i|-1$. Since C_{σ_i} is a continuous operator on $H^{\infty}_{\alpha,0}$, C_{σ_i} has dense range on $H^{\infty}_{\alpha,0}$, thus $X_i=C_{\sigma_i}(P_B)$ is dense by Corollary 2.5. Noticeing that in either case the selection of σ_i ensure that whenever φ_i is hyperbolic, the repelling fixed point $\sigma_i(\gamma_i)\in C\backslash D$ for the map $\psi_i=\sigma_i\circ\varphi_i\circ\sigma_i^{-1}\in LEF(D)$ lies in the same component $C\backslash L_i$ as D does, where L_i is the tangent line to ∂D at β_i. So regardless φ_i is a hyperbolic non-automorphism or a (parabolic or hyperbolic) automorphism, the set

$$\Delta_i=\bigcup_{n=0}^{\infty}\psi_i^{[-n]}(D)$$

is a bounded disk.

For $1\leqslant i\leqslant N$ and $n\in\mathbf{N}$, we define $S_{i,n}: X_i\to H^{\infty}_{\alpha,0}: S_{i,n}(g)=g\circ\varphi_i^{[-n]}$, where $g\in X_i(1\leqslant i\leqslant N)$. It is easy to see that $S_{i,n}$ is well defined. Also, $C^n_{\varphi_i}\circ S_{i,n}=Id_{X_i}$ pointwise on $X_i(1\leqslant i\leqslant N)$.

Let $g=f\circ\sigma_i\in X_i$, where $f\in P_B$,

$$\|S_{i,n}g\|_\alpha=\|g\circ\varphi_i^{[-n]}\|_\alpha=$$
$$\|f\circ\sigma_i\circ\varphi_i^{[-n]}\|_\alpha\to\|f\circ\sigma_i(\gamma_i)\|_\alpha=0.$$

That is $S_{i,n}\to 0$ pointwise on $X_i(1\leqslant i\leqslant N)$. When $j\neq l$, for $1\leqslant j\leqslant N$, we have

$$\|C^k_{\varphi_j}C^k_{\varphi_i}g\|_\alpha=\|p\circ\sigma_i\circ\varphi_i^{[-k]}\circ\varphi_j^{[k]}\|_\alpha\to$$
$$|p\circ\sigma_i(\gamma_i)\|_\alpha=0.$$

Thus, $C_{\varphi_1},\cdots,C_{\varphi_N}$ satisfy the d-hypercyclicity criterion with respect to sequence (k), they are d-mixing. In particular, $T_1,\cdots,T_N\in L(X)$ are d-hypercyclic.

Theorem 2.10 Let $N\geqslant 2$, $\alpha>0$, and $1<p<\infty$. For each $i=1,2,\cdots N$, let $\varphi_i\in LET(D)$ be either a parabolic automorphism or a hyperbolic map without fixed points in D. Suppose that there are no two symbols φ_i,φ_j having the same attractive fixed point β such that $\varphi_i^{-1}(\beta)=\varphi_j^{-1}(\beta)<1$. Then the operators $C_{\varphi_1},\cdots,C_{\varphi_N}$ are d-supercyclic on $H^{\infty}_{\alpha,0}$

Proof Define the set X_0 and X_1 the set as in Theorem 2.9. For each $1\leqslant i\leqslant N$, $n\in\mathbf{N}$, we define $S_{i,n}: X_i\to H^{\infty}_{\alpha,0}: S_{i,n}(g)=g\circ\varphi_i^{[-n]}$, where $g\in X_i(1\leqslant i\leqslant N)$, It is easy to see that $S_{i,n}$ is well defined. Also $C^n_{\varphi_i}\circ S_{i,n}=Id_{X_i}$ pointwise on $X_i(1\leqslant i\leqslant N)$. For $1\leqslant i\leqslant N$, let $f=p\circ\sigma_i\in X_i$, then

$$\|C^k_{\varphi_j}S_{i,k}g\|_\alpha=\|C^k_{\varphi_j}C^{-k}_{\varphi_i}(p\circ\sigma_i)\|_\alpha\to$$
$$\|p\circ\sigma_i\circ\varphi_i^{-k}\circ\varphi_j^{-k}\|_\alpha\to\|p\circ\sigma_i(\gamma_i)\|_\alpha=0.$$

For any $P_0\in X_0$ and $f_j\in X_j$, there exists a polynomial p vanishing at $\beta_i,\sigma_i(\gamma_i)$ such that $f_j=p\circ\sigma_j\in X_j$. We have

$$\lim_{k\to\infty}\|C^k_{\varphi_i}p_0\|_\alpha\|\sum_{j=1}^{N}S_{j,n_k}f_j\|_\alpha=$$
$$\lim_{k\to\infty}\|p_0\circ\varphi_i^k\|_\alpha\|\sum_{j=1}^{N}p\circ\sigma_j\circ\varphi_j^k\|_\alpha=0.$$

Since $C_{\varphi_1},\cdots,C_{\varphi_N}$ satisfy the d-Supercyclicity Criterion with respect to sequence (k), they are d-supercyclic.

Corollary 2.11 Let $N\geqslant 2$, $\alpha>0$. and $1<p<\infty$. For each $i=1,2,\cdots N$, let $\varphi_i\in LET(D)$ be either a parabolic automorphism or a hyperbolic map without fixed points in D. Suppose that there are no two symbols φ_i,φ_j having same attractive fixed point β such that $\varphi_i^{-1}(\beta)=\varphi_j^{-1}(\beta)<1$. The following statements are equivalent:

(1) The operators $C_{\varphi_1},\cdots,C_{\varphi_N}$ d-hypercyclic on $H^{\infty}_{\alpha,0}$;

(2) The operators $C_{\varphi_1},\cdots,C_{\varphi_N}$ d-mixing on $H^{\infty}_{\alpha,0}$;

(3) The operators $C_{\varphi_1},\cdots,C_{\varphi_N}$ d-supercyclic on $H^{\infty}_{\alpha,0}$.

3 Hypercyclicity of weighted composition operators on on $H_{\alpha,0}^{\infty}$

In this section, we discuss the hypercyclicity of weighted composition operator.

Lemma 3.1 Let φ be a parabolic automorphism or hyperbolic automorphism, φ^{-1} is the inverse mapping of φ. Then we have the following conclusion:

(1) if φ is a parabolic, then its fixed point z_0 lies in ∂D, and $\varphi_n(z)\to z_0$, $\varphi_n^{-1}(z)\to z_0$, for all $z\in\overline{D}$;

(2) if φ is a hyperbolic, then its has distinct fixed point z_0 and z_1 in ∂D such that $\varphi_n(z)\to z_0$ for all $z\in\overline{D}\backslash z_1$ and $\varphi_n^{-1}(z)\to z_1$ for all $z\in\overline{D}\backslash z_0$.

Theorem 3.2 If $\alpha>0$, φ is parabolic automorphism or hyperbolic automorphism, If

$$\sup_{z\in D}\prod_{i=1}^{n}|u(\varphi_i(z))|<C,$$

then uC_φ is hypercyclic.

Proof According to Lemma 3.1, there are $z_0, z_1\in\partial D$ (possibly with $z_0=z_1$) such that $\varphi_n(z)\to z_0$ for all $z\in\overline{D}\backslash z_1$ and $\varphi_n^{-1}(z)\to z_1$ for all $z\in\overline{D}\backslash z_0$. By Lemma 2.4, A_{z_0} and A_{z_1} are dense in $H_{\alpha,0}^{\infty}$.

Since $\lim\limits_{k\to\infty}f(\varphi_k(z))=0$ for $f\in A_{z_0}$ and $z\in K\subset D$, here K is any compact subsets of D, for any $\varepsilon>0$, there exists a positive number N, such that

$$|f(\varphi_{n_k}(z))|<\frac{\varepsilon}{2C}\text{ where } C=\sup_{z\in\overline{D}}|f(z)|.$$

Since $\lim\limits_{r\to 1}(1-r^2)^\alpha=0$, we can choose r such that

$$(1-r^2)^\alpha M<\frac{\varepsilon}{2C}.$$

Choose $n_k\geqslant N$. For any $f\in A_{z_0}$, we have

$$\|(uC_\varphi)^{n_k}f\|_\alpha=\|\prod_{i=1}^{n_k-1}(u(\varphi_i(z))f(\varphi_{n_k}))\|_\alpha=$$

$$\sup_{z\in D}(1-|z|^2)^\alpha|\prod_{i=1}^{n_k-1}(u(\varphi_i(z))f(\varphi_{n_k}))|\leqslant$$

$$C\sup_{|z|<r}(1-|z|^2)^\alpha|f(\varphi_{n_k}(z))|+$$

$$\sup_{|z|>r}(1-|z|^2)^\alpha|f(\varphi_{n_k}(z))|=\frac{\varepsilon}{2}+\frac{\varepsilon}{2C}.$$

Define a sequence of linear maps $S_{n,k}:A_{z_1}\to H_{\alpha,0}^{\infty}$ by

$$S_{n,k}=\prod_{i=1}^{n_k}\frac{1}{u(\varphi_i^{-1}(z))}f(\varphi_{n_k}^{-1}(z))$$

for every $z\in D$. Then, for every $f\in A_{z_1}$, it follows that

$$\|S_{n_k}f\|\to 0, (uC_\varphi)^{n_k}S_{n_k}f=f.$$

Thus uC_φ satisfy the hypercyclicity criterion, so that uC_φ is hypercyclic.

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