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一类变时滞模糊神经网络系统解的渐近概周期性

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摘 要：本文给出了一类变时滞模糊神经网络系统解的概周期性和全局指数稳定性结果及该系统渐近概周期解的具体形式.

关键词：模糊神经网络；渐近概周期；全局指数稳定；变时滞
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Asymptotical almost periodicity of solutions of a class of fuzzy cellular neural networks with varying time-delays

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Abstract: In this paper, we give some results on both asymptotical almost periodicity and global exponential stability of the solutions of a class of fuzzy cellular neural networks with time-varying delays. The concrete forms of the asymptotical almost solutions of this system are presented.

Keywords: Fuzzy cellular neural network; Asymptotical almost periodicity; Global exponential stability; Time-varying delay
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1 Introduction

The traditional cellular neural networks (CNNs) proposed by Chua and Yang[1] have been widely developed (see Refs. [1-3] and the references therein). Based on CNNs, Yang[4] introduced the fuzzy cellular neural networks (FCNNs), which added fuzzy logic to the structure of traditional CNNs. The periodicity and almost periodicity of CNNs and FCNNs have been paid great attention in the past decade due to their potential application in classification, associative memory parallel computation and other fields (see, *e.g.*, Refs. [5-10] and the references therein).

There are few works of the almost periodicity for FCNNs with delays. Let us give a brief summary in this line. Huang[11-12] studied the almost periodicity for FCNNs with time-varying delays and multi-proportional delays. Xu and Chen[13] presented some results on the almost periodicity for FCNNs with time-varying delays in leakage terms. Liang, Qian and Liu[14] studied pseudo almost periodic solutions for FCNNs with multi-proportional delays. To the best of our knowledge, there is no result on the asymptotical almost periodicity of the solutions for FCNNs with time-varying delays.

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In this paper, we consider the asymptotical almost periodicity and global exponential stability of the following FCNNs systems with time-varying delays:

$$\dot{x}_i(t)=-a_i(t)x_i(t)+\sum_{j=1}^{n} b_{ij}(t)f_j(x_j(t))+\sum_{j=1}^{n} c_{ij}(t)d_j(t)+\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+ \bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+\bigwedge_{j=1}^{n}H_{ij}(t)d_j(t)+\bigvee_{j=1}^{n}G_{ij}(t)d_j(t)+I_i(t), \quad t\geqslant t_0\geqslant 0, i\in J=\{1,2,\cdots,n\} \tag{1}$$

where $x_i(t)$ is the ith neuron's state, $a_i(t)$ is the ith neuron's self-inhibition, $b_{ij}(t)$ and $c_{ij}(t)$ are feedback template and feedforward template, $d_j(t)$ is the ith neuron's input, $\wedge$ and $\vee$ are the fuzzy AND and fuzzy OR operations, $\alpha_{ij}(t)$ and $\beta_{ij}(t)$ are the elements of the fuzzy feedback MIN template and fuzzy feedback MAX template, $\tau_{ij}\geqslant 0$ is transmission delay, $f_j(x)$ and $g_j(x)$ are activation functions, $H_{ij}(t)$ and $G_{ij}(t)$ are the elements of the fuzzy feedforward MIN template and fuzzy feedforward MAX template and $I_i(t)$ is the time-varying external input of the ith neuron. We present some results on both global exponential stability and asymptotical almost periodicity of the solutions for (1) (Theorem 3.2 and 3.3) and get the structure of the solutions for (1) (Corollary 3.4).

The initial conditions of system (1) are of the form

$$x_i(t)=\varphi_i(t), t\in[t_0-\tau_i, t_0], i\in J \tag{2}$$

where $\tau_i=\max\limits_{i,j\in J}\tau_{ij}^*$, $\tau_{ij}^*=\sup\limits_{t\in\mathbf{R}}\tau_{ij}(t)$, $\varphi_i(t)$ is continuous on $[t_0-\tau_i, t_0]$.

2 Preliminaries

The norms on $\mathbf{R}^n$ is given by $\|x\|=\max\limits_{i\in J}|x_i|$ for $x=(x_1,x_2,\cdots,x_n)\in\mathbf{R}^n$. $BC(\mathbf{R},\mathbf{R}^n)$ denotes the Banach space of bounded and continuous functions from $\mathbf{R}$ to $\mathbf{R}^n$ with supremum norm $\|f\|=\sup_{t\in\mathbf{R}}\|f(t)\|$. Even though the notation $\|\cdot\|$ is used for norms in different spaces, no confusion should arise.

Definition 2.1[15] A set $S\subset\mathbf{R}$ is said to be relatively dense if there exists $L>0$ such that $[a, a+L]\cap S\neq\varnothing$ for all $a\in\mathbf{R}$. A function $u\in BC(\mathbf{R},\mathbf{R}^n)$ is said to be almost periodic on $\mathbf{R}$ if for any $\varepsilon>0$, the set $T(u,\varepsilon)=\{\tau: \|u(t+\tau)-u(t)\|<\varepsilon, t\in\mathbf{R}\}$ is relatively dense. Denote by $AP(\mathbf{R}^n)$ the space of almost periodic functions with supremum norm.

Definition 2.2[15] A set $S\subset AP(\mathbf{R}^n)$ is a uniformly almost periodic set if it is uniformly bounded, and if given $\varepsilon>0$, then $T(S,\varepsilon)=\bigcap\limits_{f\in S}T(f,\varepsilon)$ is relatively dense and includes an interval about 0.

Definition 2.3[15] Let φ be defined on $\mathbf{R}^+=[0,+\infty)$ to $\mathbf{R}^n$. Then the continuous funciton φ is asymptotically almost periodic (abbr. a. a. p.) if and only if there is an almost periodic funciton p and a continuous fucntion q defined on $\mathbf{R}^+$ with $\lim_{t\to\infty}\|q(t)\|=0$ such that $\varphi=p+q$ on $\mathbf{R}^+$. The function p is called the almost periodic part.

Lemma 2.4[15] (i) Any finite set of functions in $AP(\mathbf{R}^n)$ is a uniformly almost periodic set.

(ii) A continuous function f is a. a. p. if and only if for every $\varepsilon>0$, there exists $T(\varepsilon)\geqslant 0$ such that $\{\tau: \sup\limits_{t\geqslant T, t+\tau\geqslant T}\|f(t+\tau)-f(t)\|<\varepsilon\}$ is relatively dense in $\mathbf{R}^+$.

Definition 2.5[11] A solution $y(t)$ of system (1) is global exponential stable if there exist two positive constants μ and M such that

$$\|y(t)-x(t)\|\leqslant M\mathrm{e}^{-\mu t}, t\geqslant t_0$$

for any solution $x(t)$ of system (1).

We will use the following assumptions:

(H_1) $a_i, b_{ij}, c_{ij}, d_j, \alpha_{ij}, \beta_{ij}, \tau_{ij}, H_{ij}, G_{ij}, I_i$ are a. a. p, $i,j\in J$;

(H_2) For $j\in J$, there exist nonnegative constants L_j^f, L_j^g such that

$$|f_j(u)-f_j(v)|\leqslant L_j^f|u-v|,$$
$$|g_j(u)-g_j(v)|\leqslant L_j^g|u-v|, u,v\in\mathbf{R};$$

(H_3) For each $i\in J$,

$$M[a_i]=\lim_{T\to+\infty}\frac{1}{T}\int_t^{t+T}a_i(s)\mathrm{d}s>0$$

and there exist a bounded and continuous function $\widetilde{a}_i:\mathbf{R}\to(0,+\infty)$ and a positive constant K_i such that

$$\mathrm{e}^{-\int_s^t a_i(u)\mathrm{d}u}\leqslant K_i\,\mathrm{e}^{-\int_s^t\widetilde{a}_i(u)\mathrm{d}u}\text{ for all }t,s\in\mathbf{R}\text{ and }t-s\geqslant 0;$$

(H_4) There exists $\eta>0$ such that $\max_{i\in J}\lambda_i(t)<-\eta<0$ for $t\geqslant t_0$, where

$$\lambda_i(t)=-a_i(t)+\sum_{j=1}^{n}(|b_{ij}(t)|L_j^f+(|\alpha_{ij}(t)|+|\beta_{ij}(t)|)L_j^g\mathrm{e}^{\frac{\eta}{2}\|\tau_{ij}\|}),i\in J.$$

Lemma 2.6[4] For $i,j\in J$, let $x_j,x_j',p_{ij},q_{ij}\in\mathbf{R}$, $k_j:\mathbf{R}\to\mathbf{R}$ are continuous functions, then

$$|\bigwedge_{j=1}^{n}p_{ij}k_j(x_j)-\bigwedge_{j=1}^{n}p_{ij}k_j(x_j')|\leqslant\sum_{j=1}^{n}|p_{ij}||k_j(x_j)-k_j(x_j')|$$

and

$$|\bigvee_{j=1}^{n}q_{ij}k_j(x_j)-\bigvee_{j=1}^{n}q_{ij}k_j(x_j')|\leqslant\sum_{j=1}^{n}|q_{ij}||k_j(x_j)-k_j(x_j')|.$$

***Remark* 1** It is not hard for us to see that the assumptions (H_1)~(H_3) and Lemma 2.6 guarantee the existence and uniqueness of solution of system (1)-(2). Here we omit the details. Similar result can be found in Ref.[13].

3 Main results

Lemma 3.1 Assume that (H_1)~(H_4) hold. Then the solution $u(t)$ of system (1)-(2) is bounded on $[t_0,+\infty)$.

Proof Let $K(t)=\max_{s\leqslant t}\|u(s)\|$. Then $\|u(t)\|\leqslant K(t)$. Denote

$$M_{i,j}=\|b_{ij}\|(|f_j(u_j(t_0))|+L_j^f|u_j(t_0)|)+(\|c_{ij}\|+\|H_{ij}\|+\|G_{ij}\|)\|d_j\|+(\|\alpha_{ij}\|+\|\beta_{ij}\|)(|g_j(u_j(t_0))|+L_j^g|u_j(t_0)|),$$

$$M=\max_i\left\{\|I_i\|+\sum_{j=1}^{n}M_{i,j}\right\}.$$

Without loss of generality, we assume that $M>0$. Then we only need to prove

$$K(t)\leqslant\max\left\{K(t_0),\frac{M}{\eta}\right\},t\geqslant t_0 \tag{3}$$

where η is given in (H_4). For $t_1\geqslant t_0$, we first prove that there exists $\delta>0$ such that

$$K(t)\leqslant\max\left\{K(t_1),\frac{M}{\eta}\right\},t\in(t_1,t_1+\delta) \tag{4}$$

If $K(t_1)=0$, (4) holds since $K(t)$ is continuous and $M/\eta>0$. So we may assume that $K(t_1)>0$, and we have two cases.

Case 1. Assume that $\|u(t_1)\|<K(t_1)$. Then $\|u(t)\|<K(t_1)$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$. So $K(t)=K(t_1)$ for $t\in(t_1,t_1+\delta)$, and (4) holds.

Case 2. Assume that $\|u(t_1)\|=K(t_1)$. By (H_2), (H_4) and Lemma 2.6, for $|u_i(t)|>0,i\in J$,

$$\begin{aligned}\left\{\frac{\mathrm{d}}{\mathrm{d}t}|u_i(t)|\right\}_{t=t_1}&=\mathrm{sign}(u_i(t_1))(-a_i(t_1)u_i(t_1)+\sum_{j=1}^{n}b_{ij}f_j(u_j(t_1))+\sum_{j=1}^{n}c_{ij}(t_1)d_j(t_1)+\\&\bigwedge_{j=1}^{n}\alpha_{ij}(t_1)g_j(u_j(t_1-\tau_{ij}(t_1)))+\bigvee_{j=1}^{n}\beta_{ij}(t_1)g_j(u_j(t_1-\tau_{ij}(t_1)))+\\&\bigwedge_{j=1}^{n}H_{ij}(t_1)d_j(t_1)+\bigvee_{j=1}^{n}G_{ij}(t_1)d_j(t_1)+I_i(t_1))\leqslant\\&-a_i(t_1)|u_i(t_1)|+\sum_{j=1}^{n}|b_{ij}(t_1)|(L_j^f|u_j(t_1)-u_j(t_0)|+|f_j(u_j(t_0))|)+\\&\sum_{j=1}^{n}(|\alpha_{ij}(t_1)|+|\beta_{ij}(t_1)|)(L_j^g|u_j(t_1-\tau_{ij}(t_1))-u_j(t_0)|+|g_j(u_j(t_0))|)+\\&\sum_{j=1}^{n}(|c_{ij}(t_1)|+|H_{ij}(t_1)|+|G_{ij}(t_1)|)|d_j(t_1)|+|I_i(t_1)|\leqslant\end{aligned}$$

$$-a_i(t_1)|u_i(t_1)|+\sum_{j=1}^{n}(|b_{ij}(t_1)|L_j^f|u_j(t_1)|+(|\alpha_{ij}(t_1)|+|\beta_{ij}(t_1)|)L_j^g|u_j(t_1-\tau_{ij}(t_1))|)+M\leqslant \lambda_i(t_1)\|u(t_1)\|+M<-\eta K(t_1)+M.$$

Thus $\left\{\frac{\mathrm{d}}{\mathrm{d}t}|u_i(t)|\right\}_{t=t_1}<0, i\in J$, if $K(t_1)\geqslant M/\eta$. So $K(t)=K(t_1), t\in(t_1,t_1+\delta)$ for some $\delta>0$, and (4) holds. If $K(t_1)<M/\eta$, $K(t)<M/\eta$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$ since $K(t)$ is continuous, and then (4) holds.

Let $\gamma=\sup\{t\geqslant t_0: K(t)\leqslant\max\{K(t_0),M/\eta\}\}$. If $\gamma\in\mathbf{R}$, we have $K(\gamma)\leqslant\max\{K(t_0),M/\eta\}$. Meanwhile, by (4), there exists $\delta>0$ such that $K(t)\leqslant\max\{K(\gamma),M/\eta\}$ for $t\in(\gamma,\gamma+\delta)$. Therefore, $K(t)\leqslant\max\{K(t_0),M/\eta\}$ for $t\in[\gamma,\gamma+\delta)$, which contradicts the definition of γ. Thus $\gamma=+\infty$, and (3) is true.

Theorem 3.2 Assume that $(H_1)\sim(H_4)$ hold. Then the solution $u(t)$ of system (1)-(2) is a. a. p. .

Proof By Lemma 3.1, $u(t)$ is bounded on $[t_0,+\infty)$. Denote

$$f_j^u=\max_{t\in[-\|u\|_{t_0},\|u\|_{t_0}]}f_j(t),$$

$$g_j^u=\max_{t\in[-\|u\|_{t_0},\|u\|_{t_0}]}g_j(t), j\in J,$$

and

$$\bar{\rho}=\max_{i,j\in J}\|\rho_{ij}\|,$$

where $\rho=c,\alpha,\beta,H,G,\tau$,

$$\bar{\zeta}=\max_{j\in J}\|\zeta_j\|,$$

where $\zeta=d,L^g,f^u,g^u$,

$$P=\{a_i,b_{ij},c_{ij},d_j,\alpha_{ij},\beta_{ij},H_{ij},G_{ij},I_i: i,j\in J\}$$

and

$$\|u\|_{t_0}=\sup_{t\in[t_0-\bar{\tau},\infty)}\|u(t)\|.$$

For $\xi=\xi^{ap}+\xi^s\in P$ with ξ^{ap} the almost periodic part and $\lim_{t\to\infty}\|\xi^s(t)\|=0$. Denote

$$P^{ap}=\{\xi^{ap}:\xi=\xi^{ap}+\xi^s\in P\},$$

$$P^s=\{\xi^s:\xi=\xi^{ap}+\xi^s\in P\}$$

and

$$\sigma=\|u\|_{t_0}+n(\bar{f}^u+2\bar{g}^u+\bar{c}+3\bar{d}+\bar{H}+\bar{G})+1 \tag{5}$$

By Lemma 2.4, P^{ap} is uniformly almost periodic for $\varepsilon>0, i,j\in J$ and $w\in T(P^{ap},\frac{\eta}{4\sigma}\varepsilon)\cap\mathbf{R}^+$. Denote

$$v(t)=u(t+w)-u(t), \psi(t)=\max_{s\leqslant t}\{\mathrm{e}^{\frac{\eta}{2}s}\|v(s)\|\}, t\geqslant t_0.$$

Let $\gamma>t_0$, such that

$$|\xi^s(t+w)-\xi^s(t)|<\frac{\eta}{4\sigma}\varepsilon, \xi^s\in P^s, t\geqslant\gamma.$$

Then

$$|\xi(t+w)-\xi(t)|\leqslant|\xi^{ap}(t+w)-\xi^{ap}(t)|+|\xi^s(t+w)-\xi^s(t)|<\frac{\eta}{2\sigma}\varepsilon, \xi\in P, t\geqslant\gamma \tag{6}$$

For $t_1\geqslant\gamma$, we claim that there exists $\delta>0$ such that

$$\psi(t)\leqslant\max\{\psi(t_1),\varepsilon e^{\frac{\eta}{2}t_1}\}, t\in(t_1,t_1+\delta) \tag{7}$$

If $\psi(t_1)=0$, (7) holds since $\psi(t)$ is continuous and $\varepsilon\,\mathrm{e}^{\frac{\eta}{2}t_1}>0$. So we may assume that $\psi(t_1)>0$, and we have the following 2 cases.

Case 1. Assume that $\mathrm{e}^{\frac{\eta}{2}t_1}\|v(t_1)\|<\psi(t_1)$. Then $\mathrm{e}^{\frac{\eta}{2}t}\|v(t)\|<\psi(t_1)$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$ since $\mathrm{e}^{\frac{\eta}{2}t}\|v(t)\|$ is continuous, and (7) holds.

Case 2. $\mathrm{e}^{\frac{\eta}{2}t_1}\|v(t_1)\|=\psi(t_1)$. Then by (H_2), Lemma 2.6, (5) and (6), for $|v_i(t)|>0, i\in J$,

$$v'_i(t)=-a_i(t+w)u_i(t+w)+a_i(t)u_i(t)+\sum_{j=1}^{n}[b_{ij}(t+w)f_j(u_j(t+w))-b_{ij}(t)f_j(u_j(t))]+\sum_{j=1}^{n}[c_{ij}(t+w)d_j(t+w)-c_{ij}(t)d_j(t)]+\bigwedge_{j=1}^{n}\alpha_{ij}(t+w)g_j(u_j(t+w-\tau_{ij}(t+w)))-\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(u_j(t-\tau_{ij}(t)))+$$

$$\bigvee_{j=1}^{n}\beta_{ij}(t+w)g_j(u_j(t+w-\tau_{ij}(t+w)))-\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(u_j(t-\tau_{ij}(t)))+$$
$$\bigwedge_{j=1}^{n}H_{ij}(t+w)d_j(t+w)-\bigwedge_{j=1}^{n}H_{ij}(t)d_j(t)+\bigvee_{j=1}^{n}G_{ij}(t+w)d_j(t+w)-$$
$$\bigvee_{j=1}^{n}G_{ij}(t)d_j(t)+I_i(t+w)-I_i(t)\leqslant$$
$$-a_i(t)v_i(t)+|a_i(t+w)-a_i(t)|\,\|u\|_{t_0}+\sum_{j=1}^{n}[|b_{ij}(t+w)-b_{ij}(t)|\bar{f}^u+|b_{ij}(t)|L_j^f|v_j(t)|]+$$
$$\sum_{j=1}^{n}[|c_{ij}(t+w)-c_{ij}(t)|\bar{d}+|d_j(t+w)-d_j(t)|\bar{c}]+$$
$$\sum_{j=1}^{n}[(|\alpha_{ij}(t+w)-\alpha_{ij}(t)|+|\beta_{ij}(t+w)-\beta_{ij}(t)|)\bar{g}^u+(|\alpha_{ij}(t)+|\beta_{ij}(t)|)L_j^g|v_j(t-\tau_{ij}(t))|]+$$
$$\sum_{j=1}^{n}[(|H_{ij}(t+w)-H_{ij}(t)|+|G_{ij}(t+w)-G_{ij}(t)|)\bar{d}+$$
$$|d_j(t+w)-d_j(t)|(\bar{H}+\bar{G})]+I_i(t+w)-I_i(t)\leqslant$$
$$-a_i(t)v_i(t)+\sum_{j=1}^{n}[|b_{ij}(t)|L_j^f|v_j(t)|+(|\alpha_{ij}(t)|+|\beta_{ij}(t)|)L_j^g|v_j(t-\tau_{ij}(t))|]+$$
$$[\|u\|_{t_0}+n(\bar{f}^u+2\bar{g}^u+\bar{c}+3\bar{d}+H+\bar{G})+1]\frac{\eta}{2\sigma}\varepsilon=$$
$$-a_i(t)v_i(t)+\sum_{j=1}^{n}[|b_{ij}(t)|L_j^f|v_j(t)|+(|\alpha_{ij}(t)|+|\beta_{ij}(t)|)L_j^g|v_j(t-\tau_{ij}(t))|]+\frac{\eta}{2}\varepsilon.$$

Noticing that

$$|v_j(t-\tau_{ij}(t))|\leqslant\psi(t-\tau_{ij}(t))e^{\frac{\eta}{2}(\tau_{ij}(t)-t)}\leqslant\psi(t)e^{\frac{\eta}{2}(\|\tau_{ij}\|-t)},t\geqslant\gamma,$$

by (H_4) we have

$$\operatorname{sign}(v_i(t))v'_i(t)\leqslant$$
$$-a_i(t)|v_i(t)|+\sum_{j=1}^{n}[|b_{ij}(t)|L_j^f+(|\alpha_{ij}(t)|+|\beta_{ij}(t)|)L_j^g e^{\frac{\eta}{2}\|\tau_{ij}\|}]\psi(t)e^{-\frac{\eta}{2}t}+\frac{\eta}{2}\varepsilon=$$
$$-a_i(t)|v_i(t)|+(\lambda_i(t)+a_i(t))\psi(t)e^{-\frac{\eta}{2}t}+\frac{\eta}{2}\varepsilon.$$

Then, for the index such that $e^{\frac{\eta}{2}t_1}|v_i(t_1)|=\psi(t_1)$,

$$\left\{\frac{d}{dt}(e^{\frac{\eta}{2}t}|v_i(t)|)\right\}_{t=t_1}=\frac{\eta}{2}e^{\frac{\eta}{2}t_1}|v_i(t_1)|+e^{\frac{\eta}{2}t_1}\operatorname{sign}(v_i(t_1))v'_i(t_1)\leqslant$$
$$\left(\frac{\eta}{2}-a_i(t_1)+\lambda_i(t_1)+a_i(t_1)\right)\psi(t_1)+e^{\frac{\eta}{2}t_1}\frac{\eta}{2}\varepsilon<-\frac{\eta}{2}\psi(t_1)+e^{\frac{\eta}{2}t_1}\frac{\eta}{2}\varepsilon.$$

Thus $\left\{\frac{d}{dt}(e^{\frac{\eta}{2}t}|v_i(t)|)\right\}_{t=t_1}<0, i\in J$ if $\psi(t_1)\geqslant\varepsilon e^{\frac{\eta}{2}t_1}$. So $\psi(t)=\psi(t_1)$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$ by the definition of $\psi(t)$, and then (7) holds.

Otherwise, if $\psi(t_1)<\varepsilon e^{\frac{\eta}{2}t_1}$, $\psi(t)<\varepsilon e^{\frac{\eta}{2}t_1}$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$ since $\psi(t)$ is continuous, and then (7) holds.

Next we prove that

$$\psi(t)\leqslant\varepsilon e^{\frac{\eta}{2}t},t\geqslant\gamma \tag{8}$$

Let $\gamma_1=\sup\{t\geqslant t_0:\psi(t)\leqslant\varepsilon e^{\frac{\eta}{2}t}\}$. If $\gamma_1\in\mathbf{R}$, we have $\psi(\gamma_1)\leqslant\varepsilon e^{\frac{\eta}{2}\gamma_1}$. Meanwhile, by (7), there exists $\delta>0$ such that $\psi(t)\leqslant\max\{\psi(\gamma_1),\varepsilon e^{\frac{\eta}{2}t}\}$ for $t\in(\gamma_1,\gamma_1+\delta)$. Therefore, $\psi(t)\leqslant\varepsilon e^{\frac{\eta}{2}t}$ for $t\in[\gamma_1,\gamma_1+\delta)$, which contradicts the definition of γ_1. Thus $\gamma_1=+\infty$, (8) is true.

Now, it follows from (8) that

$$\|u(t+w)-u(t)\|=\|v(t)\|\leqslant$$
$$e^{-\frac{\eta}{2}t}\psi(t)\leqslant e^{-\frac{\eta}{2}t}\varepsilon e^{\frac{\eta}{2}t}=\varepsilon,t\geqslant\gamma.$$

This implies that

$$A=\{w:\sup_{t\geqslant\gamma,t+w\geqslant\gamma}\|u(t+w)-u(t)\|<2\varepsilon\}\supset$$
$$T\left(P^{ap},\frac{\eta}{4\sigma}\varepsilon\right)\cap\mathbf{R}^+,$$

which means that A is relatively dense in $\mathbf{R}^+$ since

$T\left(P^{ap},\frac{\eta}{4\sigma}\varepsilon\right)$ is relatively dense in $\mathbf{R}^+$. Then u is a. a. p. by Lemma 2. 4 (ii).

Theorem 3. 3 Assume that (H_1) ~ (H_4) hold. Then any solution of system (1) is global exponential stable.

Proof Let $u(t), x(t)$ are two solutions of (1), $w(t)=u(t)-x(t), t\in[t_0,+\infty)$. It suffices to prove that there exist two positive constants μ and M such that

$$\| w(t) \| \leqslant M e^{-\mu t}, t\geqslant t_0 \tag{9}$$

Let $\theta(t)=\max\limits_{s\leqslant t}\{e^{\eta s/2}\| w(s) \|\}$. If $\theta(t)=0$ for $t\in[t_0,+\infty)$, (9) holds. If $\theta(t)=0$, then $\theta(s)=0$ for $s\in[t_0,t]$. If $\theta(t)>0$, then $\theta(s)>0$ for $s\geqslant t$. So to prove (9), we only need to consider $t\in[t_0,\infty)$ such that $\theta(t)>0$. Without loss of generality, we may assume that $\theta(t)>0, t\in[t_0,\infty)$.

Let $t_1\geqslant t_0$ with $\theta(t_1)>0$. We claim that, for some $\delta>0$,

$$\theta(t)=\theta(t_1), t\in(t_1,t_1+\delta) \tag{10}$$

If $e^{\eta t_1/2}\| w(t_1) \|<\theta(t_1)$, then $e^{\eta t/2}\| w(t) \|<\theta(t_1)$ for $t\in(t_1,t_1+\delta)$ with some $\delta>0$ since $e^{\eta t/2}\| w(t) \|$ is continuous, and (10) holds. If $e^{\eta t_1/2}\| w(t_1) \|=\theta(t_1)$, for $|w_i(t_1)|>0, i\in J$, then

$$\begin{aligned}
&\left\{\frac{d}{dt}(e^{\eta t/2}|w_i(t)|)\right\}_{t=t_1}=\frac{\eta}{2}e^{\eta t_1/2}|w_i(t_1)|+e^{\eta t_1/2}\mathrm{sign}(w_i(t_1))\{-a_i(t_1)w_i(t_1)+\\
&\sum_{i=1}^{n}b_{ij}(t_1)[f_j(u_j(t_1))-f_j(x_j(t_1))]+\bigwedge_{j=1}^{n}\alpha_{ij}(t_1)[g_j(u_j(t_1-\tau_{ij}(t_1)))-g_j(x_j(t_1-\tau_{ij}(t_1)))]+\\
&\bigvee_{j=1}^{n}\beta_{ij}(t_1)[g_j(u_j(t_1-\tau_{ij}(t_1)))-g_j(x_j(t_1-\tau_{ij}(t_1)))]\}\leqslant\\
&e^{\eta t_1/2}\{-(a_i(t_1)-\eta/2)\ |w_i(t_1)|+\sum_{j=1}^{n}|b_{it_1j}(t_1)|L_j^f|w_j(t_1)|+\\
&\sum_{j=1}^{n}(|\alpha_{ij}(t_1)|+|\beta_{ij}(t_1)|)L_j^g\ |w_j(t_1-\tau_{ij}(t_1))|\}\leqslant\\
&\left\{-\left(a_i(t_1)-\frac{\eta}{2}\right)+\sum_{j=1}^{n}[|b_{ij}(t_1)|L_j^f+(|\alpha_{ij}(t_1)|+|\beta_{ij}(t_1)|)L_j^g e^{\|\tau_{ij}\|\frac{\eta}{2}}]\right\}e^{\frac{\eta t_1}{2}}\ \| w(t_1) \|\leqslant\\
&-\frac{\eta}{2}\theta(t_1)<0.
\end{aligned}$$

This simply implies (10) holds for some $\delta>0$. It follows from (10) that $\theta(t)\leqslant\theta(t_0)$ for $t\geqslant t_0$. Then $\| w(t) \|\leqslant\theta(t_0)e^{-\eta t/2}$ for all $t\geqslant t_0$. This follows that (9) holds with $M=\theta(t_0), \mu=\eta/2$.

By Theorem 3. 2 and 3. 3, we have the following corollary immediately.

Corollary 3. 4 Assume that (H_1) ~ (H_4) hold. Then any solution of (1) is a. a. p., and there exists a function $p\in AP(\mathbf{R}^n)$ such that any solution of system (1) has the form $p+q$ with $\lim\limits_{t\to+\infty}q(t)=0$.

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